The modulus of the complex number $a + bi$ is $|a + bi| = \sqrt{a^2 + b^2}$. This is the distance between the origin $(0, 0)$ and the point $(a, b)$ in the complex plane. For two points in the complex plane, the distance between the points is the modulus of the difference of the two complex numbers.

Let $(a, b)$ and $(s, t)$ be points in the complex plane. The difference of the complex numbers is $(s + ti) - (a + bi) = (s - a) + (t - b)i$. The modulus of the difference is $|(s - a) + (t - b)i| = \sqrt{(s - a)^2 + (t - b)^2}$.

So, $d = \sqrt{(s - a)^2 + (t - b)^2}$ is the difference between the two points in the complex plane.

**Distance Formula in the Complex Plane**

The difference between the points $(a, b)$ and $(s, t)$ in the complex plane is $d = \sqrt{(s - a)^2 + (t - b)^2}$.

**EXAMPLE 1**

**Finding the Distance Between Points in the Complex Plane**

Find the distance between the points $2 + 3i$ and $5 - 2i$ in the complex plane.

**Solution**

Let $a + bi = 2 + 3i$ and $s + ti = 5 - 2i$. The difference between the complex numbers is

$$(5 - 2i) - (2 + 3i) = (5 - 2) + (-2 - 3)i = 3 - 5i.$$ 

The distance is

$$d = \sqrt{3^2 + (-5)^2} = \sqrt{34} \approx 5.83 \text{ units}$$

as shown in the figure below.
The midpoint of the line segment joining two complex numbers $a + bi$ and $s + ti$ is the average of the numbers at the endpoints.

**Midpoint Formula in the Complex Plane**

The Midpoint Formula is

$$
\text{Midpoint} = \frac{a + s}{2} + \left(\frac{b + t}{2}\right)i.
$$

**EXAMPLE 2** Finding the Midpoint of a Line Segment in the Complex Plane

Find the midpoint of the line segment joining the points $4 - 3i$ and $2 + 2i$.

**Solution**

Let $a + bi = 4 - 3i$ and $s + ti = 2 + 2i$. Apply the Midpoint Formula.

$$
\text{Midpoint} = \frac{a + s}{2} + \left(\frac{b + t}{2}\right)i = \frac{4 + 2}{2} + \left(\frac{-3 + 2}{2}\right)i = 3 - 0.5i
$$

The midpoint of the line segment joining the points $4 - 3i$ and $2 + 2i$ is $3 - 0.5i$, as shown in the figure below.

**Exercises**

**Finding the Distance Between Points in the Complex Plane**

In Exercises 1–4, find the distance between the points in the complex plane.

1. $1 + 2i$, $-1 + 4i$
2. $-5 + i$, $-2 + 5i$
3. $6i$, $3 - 4i$
4. $-7 - 3i$, $3 + 5i$

**Finding the Midpoint of a Line Segment in the Complex Plane**

In Exercises 5–8, find the midpoint of the line segment joining the points.

5. $2 + i$, $6 + 5i$
6. $-3 + 4i$, $1 - 2i$
7. $7i$, $9 - 10i$
8. $-1 - \frac{3}{4}i$, $\frac{1}{2} + \frac{1}{4}i$