

# Appendix B: Review of Graphs, Equations, and Inequalities

## B.1 The Cartesian Plane

### The Cartesian Plane

Just as you can represent real numbers by points on a real number line, you can represent ordered pairs of real numbers by points in a plane called the **rectangular coordinate system**, or the **Cartesian plane**, after the French mathematician René Descartes (1596–1650).

The Cartesian plane is formed by using two real number lines intersecting at right angles, as shown in Figure B.1. The horizontal real number line is usually called the **x-axis**, and the vertical real number line is usually called the **y-axis**. The point of intersection of these two axes is the **origin**, and the two axes divide the plane into four parts called **quadrants**.

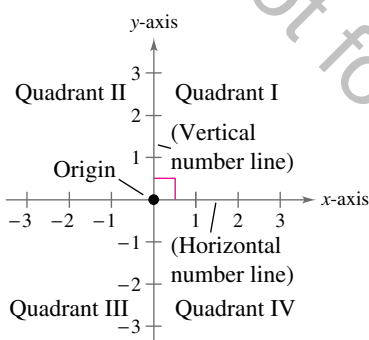


Figure B.1 The Cartesian Plane

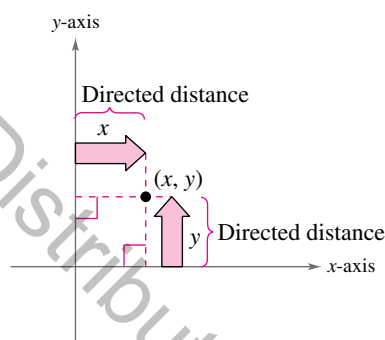
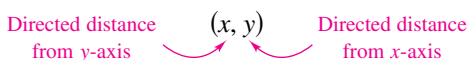


Figure B.2 Ordered Pair  $(x, y)$

Each point in the plane corresponds to an **ordered pair**  $(x, y)$  of real numbers  $x$  and  $y$ , called **coordinates** of the point. The **x-coordinate** represents the directed distance from the  $y$ -axis to the point, and the **y-coordinate** represents the directed distance from the  $x$ -axis to the point, as shown in Figure B.2.



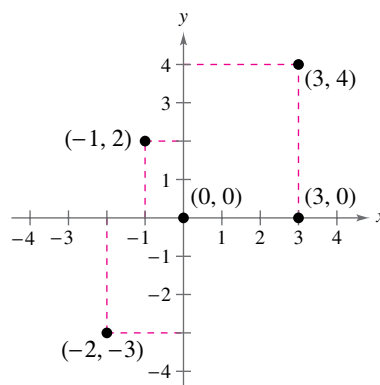
The notation  $(x, y)$  denotes both a point in the plane and an open interval on the real number line. The context will tell you which meaning is intended.

### EXAMPLE 1 Plotting Points in the Cartesian Plane

Plot the points  $(-1, 2)$ ,  $(3, 4)$ ,  $(0, 0)$ ,  $(3, 0)$ , and  $(-2, -3)$  in the Cartesian plane.

#### Solution

To plot the point  $(-1, 2)$ , imagine a vertical line through  $-1$  on the  $x$ -axis and a horizontal line through  $2$  on the  $y$ -axis. The intersection of these two lines is the point  $(-1, 2)$ . This point is one unit to the left of the  $y$ -axis and two units up from the  $x$ -axis. The other four points can be plotted in a similar way, as shown in the figure.



#### What you should learn

- ▶ Plot points in the Cartesian plane and sketch scatter plots.
- ▶ Use the Distance Formula to find the distance between two points.
- ▶ Use the Midpoint Formula to find the midpoint of a line segment.
- ▶ Find the equation of a circle.
- ▶ Translate points in the plane.

#### Why you should learn it

The Cartesian plane can be used to represent relationships between two variables. For instance, Exercise 96 on page B11 shows how to represent graphically the numbers of performers elected to the Rock and Roll Hall of Fame from 1995 through 2014.

The beauty of a rectangular coordinate system is that it enables you to see relationships between two variables. It would be difficult to overestimate the importance of Descartes’s introduction of coordinates in the plane. Today, his ideas are in common use in virtually every scientific and business-related field.

In the next example, data are represented graphically by points plotted in a rectangular coordinate system. This type of graph is called a **scatter plot**.

**EXAMPLE 2** Sketching a Scatter Plot

The numbers of employees  $E$  (in thousands) in dentist offices in the United States from 2004 through 2013 are shown in the table, where  $t$  represents the year. Sketch a scatter plot of the data by hand. (Source: U.S. Bureau of Labor Statistics)

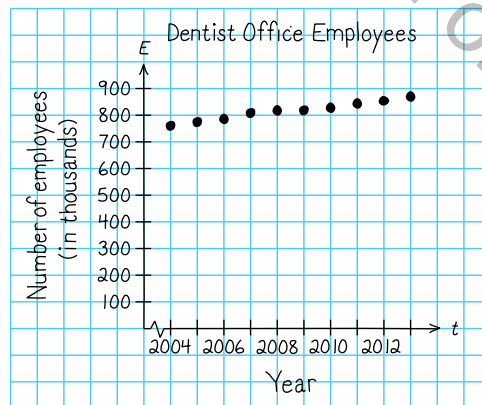
Year, $t$	Employees, $E$
2004	760
2005	774
2006	786
2007	808
2008	818
2009	818
2010	828
2011	844
2012	854
2013	870

**Solution**

Before you sketch the scatter plot, it is helpful to represent each pair of values by an ordered pair  $(t, E)$  as follows.

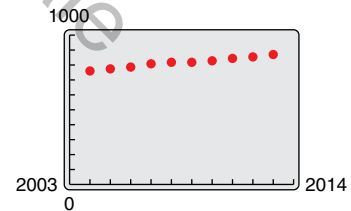
- $(2004, 760)$ ,  $(2005, 774)$ ,  $(2006, 786)$ ,  $(2007, 808)$ ,  $(2008, 818)$ ,
- $(2009, 818)$ ,  $(2010, 828)$ ,  $(2011, 844)$ ,  $(2012, 854)$ ,  $(2013, 870)$

To sketch a scatter plot of the data shown in the table, first draw a vertical axis to represent the number of employees (in thousands) and a horizontal axis to represent the year. Then plot the resulting points, as shown in the figure. Note that the break in the  $t$ -axis indicates that the numbers 0 through 2003 have been omitted.



**Technology Tip**

You can use a graphing utility to graph the scatter plot in Example 2. For instructions on how to use the *list editor* and the *statistical plotting* feature, see Appendix A; for specific keystrokes, go to this textbook’s *Companion Website*.



In Example 2, you could have let  $t = 1$  represent the year 2004. In that case, the horizontal axis of the graph would not have been broken, and the tick marks would have been labeled 1 through 10 (instead of 2004 through 2013).

## The Distance Formula

Recall from the Pythagorean Theorem that, for a right triangle with hypotenuse of length  $c$  and sides of lengths  $a$  and  $b$ , you have

$$a^2 + b^2 = c^2 \quad \text{Pythagorean Theorem}$$

as shown in Figure B.3. (The converse is also true. That is, if  $a^2 + b^2 = c^2$ , then the triangle is a right triangle.)

Suppose you want to determine the distance  $d$  between two points  $(x_1, y_1)$  and  $(x_2, y_2)$  in the plane. Using these two points, you can form a right triangle, as shown in Figure B.4. The length of the vertical side of the triangle is  $|y_2 - y_1|$  and the length of the horizontal side is  $|x_2 - x_1|$ . By the Pythagorean Theorem,

$$\begin{aligned} d^2 &= |x_2 - x_1|^2 + |y_2 - y_1|^2 \\ d &= \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2} \\ d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}. \end{aligned}$$

This result is called the **Distance Formula**.

### The Distance Formula

The distance  $d$  between the points  $(x_1, y_1)$  and  $(x_2, y_2)$  in the plane is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

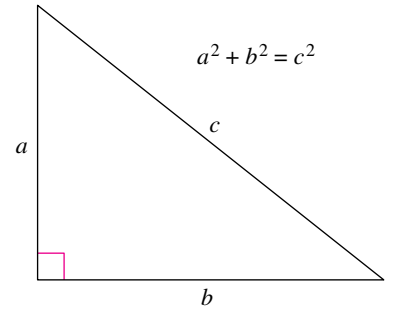


Figure B.3

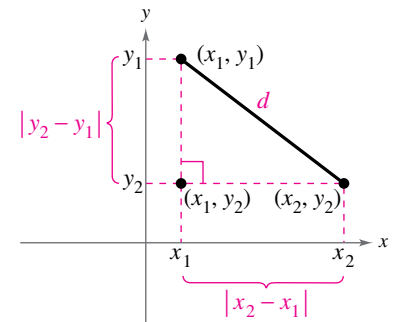


Figure B.4

### EXAMPLE 3 Finding a Distance

Find the distance between the points

$$(-2, 1) \text{ and } (3, 4).$$

#### Algebraic Solution

Let  $(x_1, y_1) = (-2, 1)$  and  $(x_2, y_2) = (3, 4)$ . Then apply the Distance Formula as follows.

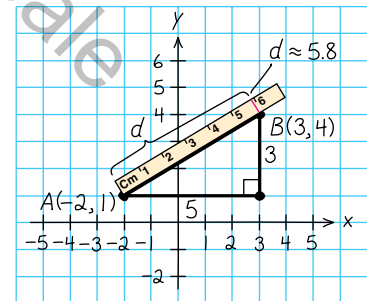
$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{[3 - (-2)]^2 + (4 - 1)^2} && \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2. \\ &= \sqrt{(5)^2 + (3)^2} && \text{Simplify.} \\ &= \sqrt{34} && \text{Simplify.} \\ &\approx 5.83 && \text{Use a calculator.} \end{aligned}$$

So, the distance between the points is about 5.83 units. You can use the Pythagorean Theorem to check that the distance is correct.

$$\begin{aligned} d^2 &\stackrel{?}{=} 3^2 + 5^2 && \text{Pythagorean Theorem} \\ (\sqrt{34})^2 &\stackrel{?}{=} 3^2 + 5^2 && \text{Substitute for } d. \\ 34 &= 34 && \text{Distance checks. } \checkmark \end{aligned}$$

#### Graphical Solution

Use centimeter graph paper to plot the points  $A(-2, 1)$  and  $B(3, 4)$ . Carefully sketch the line segment from  $A$  to  $B$ . Then use a centimeter ruler to measure the length of the segment.



The line segment measures about 5.8 centimeters, as shown in the figure. So, the distance between the points is about 5.8 units.



When the Distance Formula is used, it does not matter which point is  $(x_1, y_1)$  and which is  $(x_2, y_2)$ , because the result will be the same. For instance, in Example 3, let  $(x_1, y_1) = (3, 4)$  and  $(x_2, y_2) = (-2, 1)$ . Then

$$d = \sqrt{(-2 - 3)^2 + (1 - 4)^2} = \sqrt{(-5)^2 + (-3)^2} = \sqrt{34} \approx 5.83.$$

**EXAMPLE 4** Verifying a Right Triangle

Show that the points  $(2, 1)$ ,  $(4, 0)$ , and  $(5, 7)$  are the vertices of a right triangle.

**Solution**

The three points are plotted in Figure B.5. Using the Distance Formula, you can find the lengths of the three sides as follows.

$$d_1 = \sqrt{(5 - 2)^2 + (7 - 1)^2} = \sqrt{9 + 36} = \sqrt{45}$$

$$d_2 = \sqrt{(4 - 2)^2 + (0 - 1)^2} = \sqrt{4 + 1} = \sqrt{5}$$

$$d_3 = \sqrt{(5 - 4)^2 + (7 - 0)^2} = \sqrt{1 + 49} = \sqrt{50}$$

Because

$$(d_1)^2 + (d_2)^2 = 45 + 5 = 50 = (d_3)^2$$

you can conclude that the triangle must be a right triangle.

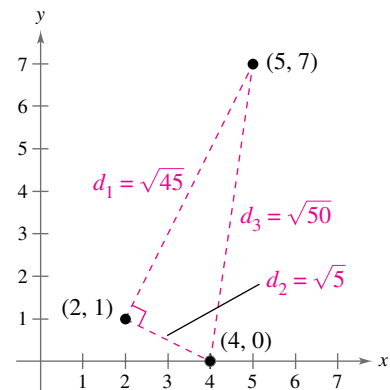


Figure B.5

**EXAMPLE 5** Finding the Length of a Pass

A football quarterback throws a pass from the 28-yard line, 40 yards from the sideline. A wide receiver catches the pass on the 5-yard line, 20 yards from the same sideline, as shown in Figure B.6. How long is the pass?

**Solution**

You can find the length of the pass by finding the distance between the points  $(40, 28)$  and  $(20, 5)$ .

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$= \sqrt{(40 - 20)^2 + (28 - 5)^2}$	Substitute for $x_1, y_1, x_2,$ and $y_2$ .
$= \sqrt{20^2 + 23^2}$	Simplify.
$= \sqrt{400 + 529}$	Simplify.
$= \sqrt{929}$	Simplify.
$\approx 30$	Use a calculator.

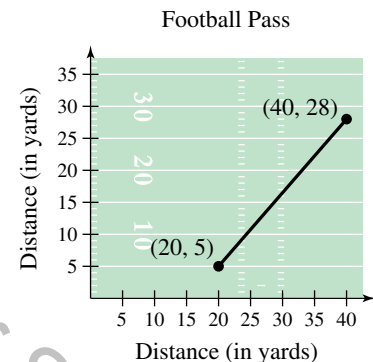


Figure B.6

So, the pass is about 30 yards long. ■

In Example 5, the scale along the goal line does not normally appear on a football field. However, when you use coordinate geometry to solve real-life problems, you are free to place the coordinate system in any way that is convenient for the solution of the problem.



## The Midpoint Formula

To find the **midpoint** of the line segment that joins two points in a coordinate plane, find the average values of the respective coordinates of the two endpoints using the **Midpoint Formula**.

### The Midpoint Formula

The midpoint of the line segment joining the points  $(x_1, y_1)$  and  $(x_2, y_2)$  is given by the Midpoint Formula

$$\text{Midpoint} = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

### EXAMPLE 6 Finding a Line Segment's Midpoint

Find the midpoint of the line segment joining the points  $(-5, -3)$  and  $(9, 3)$ .

#### Solution

Let  $(x_1, y_1) = (-5, -3)$  and  $(x_2, y_2) = (9, 3)$ .

$$\begin{aligned} \text{Midpoint} &= \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) && \text{Midpoint Formula} \\ &= \left( \frac{-5 + 9}{2}, \frac{-3 + 3}{2} \right) && \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2. \\ &= (2, 0) && \text{Simplify.} \end{aligned}$$

The midpoint of the line segment is  $(2, 0)$ , as shown in Figure B.7.

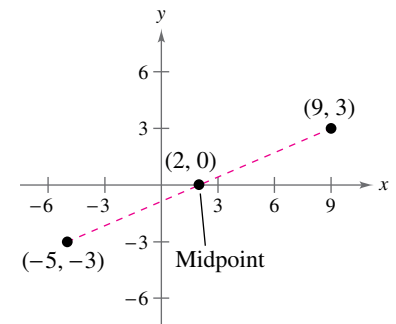


Figure B.7

### EXAMPLE 7 Estimating Annual Revenues

Verizon Communications had annual revenues of \$110.9 billion in 2011 and \$120.6 billion in 2013. Without knowing any additional information, what would you estimate the 2012 revenue to have been? (Source: Verizon Communications)

#### Solution

One solution to the problem is to assume that revenue followed a linear pattern. With this assumption, you can estimate the 2012 revenue by finding the midpoint of the line segment connecting the points  $(2011, 110.9)$  and  $(2013, 120.6)$ .

$$\begin{aligned} \text{Midpoint} &= \left( \frac{2011 + 2013}{2}, \frac{110.9 + 120.6}{2} \right) \\ &= (2012, 115.75) \end{aligned}$$

So, you would estimate the 2012 revenue to have been about \$115.75 billion, as shown in Figure B.8. (The actual 2012 revenue was \$115.8 billion.)

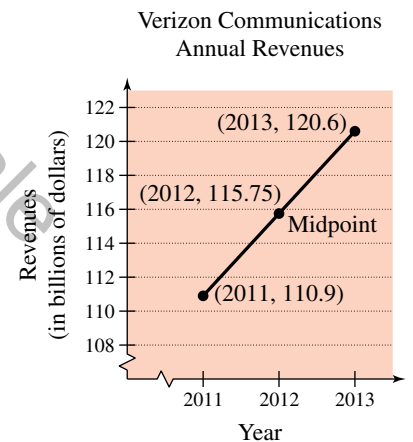


Figure B.8

## The Equation of a Circle

The Distance Formula provides a convenient way to define circles. A circle of radius  $r$  with center at the point  $(h, k)$  is shown in Figure B.9. The point  $(x, y)$  is on this circle if and only if its distance from the center  $(h, k)$  is  $r$ . This means that a **circle** in the plane consists of all points  $(x, y)$  that are a given positive distance  $r$  from a fixed point  $(h, k)$ . Using the Distance Formula, you can express this relationship by saying that the point  $(x, y)$  lies on the circle if and only if

$$\sqrt{(x - h)^2 + (y - k)^2} = r.$$

By squaring each side of this equation, you obtain the **standard form of the equation of a circle**.

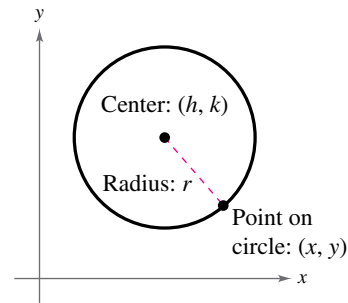


Figure B.9

### Standard Form of the Equation of a Circle

The **standard form of the equation of a circle** is

$$(x - h)^2 + (y - k)^2 = r^2. \quad \text{Circle with center at } (h, k)$$

The point  $(h, k)$  is the **center** of the circle, and the positive number  $r$  is the **radius** of the circle. The standard form of the equation of a circle whose center is the origin,  $(h, k) = (0, 0)$ , is

$$x^2 + y^2 = r^2. \quad \text{Circle with center at origin}$$

### EXAMPLE 8 Writing an Equation of a Circle

The point  $(3, 4)$  lies on a circle whose center is at  $(-1, 2)$ , as shown in Figure B.10. Write the standard form of the equation of this circle.

#### Solution

The radius  $r$  of the circle is the distance between  $(-1, 2)$ , and  $(3, 4)$ .

$$\begin{aligned} r &= \sqrt{(x - h)^2 + (y - k)^2} && \text{Distance Formula} \\ &= \sqrt{[3 - (-1)]^2 + (4 - 2)^2} && \text{Substitute for } x, y, h, \text{ and } k. \\ &= \sqrt{4^2 + 2^2} && \text{Simplify.} \\ &= \sqrt{16 + 4} && \text{Simplify.} \\ &= \sqrt{20} && \text{Radius} \end{aligned}$$

Using  $(h, k) = (-1, 2)$  and  $r = \sqrt{20}$ , the equation of the circle is

$$\begin{aligned} (x - h)^2 + (y - k)^2 &= r^2 && \text{Equation of circle} \\ [x - (-1)]^2 + (y - 2)^2 &= (\sqrt{20})^2 && \text{Substitute for } h, k, \text{ and } r. \\ (x + 1)^2 + (y - 2)^2 &= 20. && \text{Standard form} \end{aligned}$$

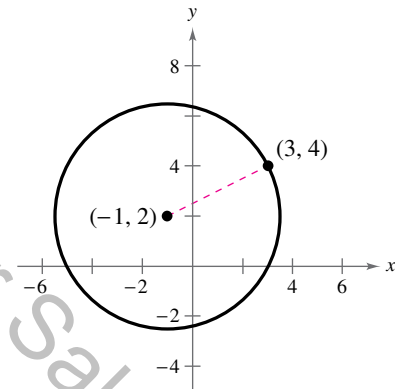


Figure B.10

Be careful when you are finding  $h$  and  $k$  from the standard form of the equation of a circle. For instance, to find  $h$  and  $k$  from the equation of the circle in Example 8, rewrite the quantities  $(x + 1)^2$  and  $(y - 2)^2$  using subtraction.

$$(x + 1)^2 = [x - (-1)]^2 \quad \text{and} \quad (y - 2)^2 = [y - (2)]^2$$

So,  $h = -1$  and  $k = 2$ .

## Application

Much of computer graphics consists of transformations of points in a coordinate plane. One type of transformation, a translation, is illustrated in Example 9. Other types of transformations include reflections, rotations, and stretches.

### EXAMPLE 9 Translating Points in the Plane

See *LarsonPrecalculus.com* for an interactive version of this type of example.

The triangle in Figure B.11 has vertices at the points  $(-1, 2)$ ,  $(1, -4)$ , and  $(2, 3)$ . Shift the triangle three units to the right and two units upward and find the vertices of the shifted triangle, as shown in Figure B.12.

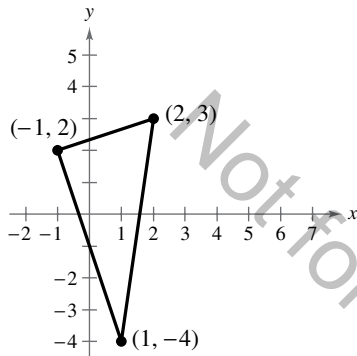


Figure B.11

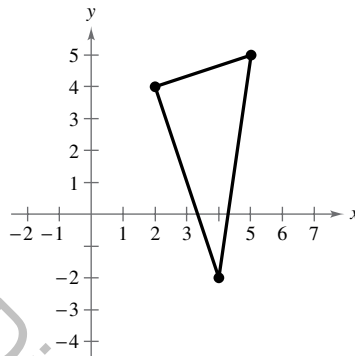


Figure B.12

### Solution

To shift the vertices three units to the right, add 3 to each of the  $x$ -coordinates. To shift the vertices two units upward, add 2 to each of the  $y$ -coordinates.

Original Point	Translated Point
$(-1, 2)$	$(-1 + 3, 2 + 2) = (2, 4)$
$(1, -4)$	$(1 + 3, -4 + 2) = (4, -2)$
$(2, 3)$	$(2 + 3, 3 + 2) = (5, 5)$

Plotting the translated points and sketching the line segments between them produces the shifted triangle shown in Figure B.12. ■

Example 9 shows how to translate points in a coordinate plane. The following transformed points are related to the original points as follows.

Original Point	Transformed Point	
$(x, y)$	$(-x, y)$	$(-x, y)$ is a reflection of the original point in the $y$ -axis.
$(x, y)$	$(x, -y)$	$(x, -y)$ is a reflection of the original point in the $x$ -axis.
$(x, y)$	$(-x, -y)$	$(-x, -y)$ is a reflection of the original point through the origin.

The figures provided with Example 9 were not really essential to the solution. Nevertheless, it is strongly recommended that you develop the habit of including sketches with your solutions because they serve as useful problem-solving tools.

**B.1 Exercises**

See *CalcChat.com* for tutorial help and worked-out solutions to odd-numbered exercises. For instructions on how to use a graphing utility, see Appendix A.

**Vocabulary and Concept Check**

In Exercises 1–4, fill in the blank(s).

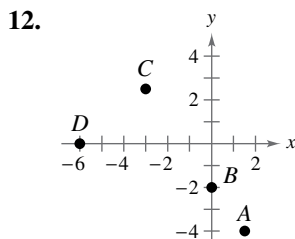
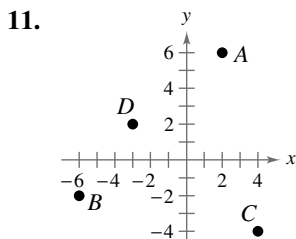
1. An ordered pair of real numbers can be represented in a plane called the rectangular coordinate system or the \_\_\_\_\_ plane.
2. The \_\_\_\_\_ is a result derived from the Pythagorean Theorem.
3. Finding the average values of the respective coordinates of the two endpoints of a line segment in a coordinate plane is also known as using the \_\_\_\_\_.
4. The standard form of the equation of a circle is \_\_\_\_\_, where the point  $(h, k)$  is the \_\_\_\_\_ of the circle and the positive number  $r$  is the \_\_\_\_\_ of the circle.

In Exercises 5–10, match each term with its definition.

- |                     |                                                                |
|---------------------|----------------------------------------------------------------|
| 5. $x$ -axis        | (a) point of intersection of vertical axis and horizontal axis |
| 6. $y$ -axis        | (b) directed distance from the $x$ -axis                       |
| 7. origin           | (c) horizontal real number line                                |
| 8. quadrants        | (d) four regions of the coordinate plane                       |
| 9. $x$ -coordinate  | (e) directed distance from the $y$ -axis                       |
| 10. $y$ -coordinate | (f) vertical real number line                                  |

**Procedures and Problem Solving**

**Approximating Coordinates of Points** In Exercises 11 and 12, approximate the coordinates of the points.



**Determining Quadrants** In Exercises 21–30, determine the quadrant(s) in which  $(x, y)$  is located so that the condition(s) is (are) satisfied.

- |                          |                          |
|--------------------------|--------------------------|
| 21. $x > 0$ and $y < 0$  | 22. $x < 0$ and $y < 0$  |
| 23. $x = -4$ and $y > 0$ | 24. $x > 2$ and $y = 3$  |
| 25. $y < -5$             | 26. $x > 4$              |
| 27. $x < 0$ and $-y > 0$ | 28. $-x > 0$ and $y < 0$ |
| 29. $xy > 0$             | 30. $xy < 0$             |

**Plotting Points in the Cartesian Plane** In Exercises 13–16, plot the points in the Cartesian plane.

13.  $(-4, 2), (-3, -6), (0, 5), (1, -4)$
14.  $(4, -2), (0, 0), (-4, 0), (-5, -5)$
15.  $(3, 8), (0.5, -1), (5, -6), (-2, -2.5)$
16.  $(1, -\frac{1}{2}), (-\frac{5}{2}, 2), (3, -3), (\frac{3}{2}, 1)$

**Finding Coordinates of Points** In Exercises 17–20, find the coordinates of the point.

17. The point is located five units to the left of the  $y$ -axis and four units above the  $x$ -axis.
18. The point is located three units below the  $x$ -axis and two units to the right of the  $y$ -axis.
19. The point is on the  $y$ -axis and six units below the  $x$ -axis.
20. The point is on the  $x$ -axis and 11 units to the left of the  $y$ -axis.

**31. Accounting** The table shows the sales  $y$  (in millions of dollars) for Apple for the years 2002 through 2013. Sketch a scatter plot of the data. (Source: Apple Inc.)

Year	Sales, $y$ (in millions of dollars)
2002	5,742
2003	6,207
2004	8,279
2005	13,931
2006	19,315
2007	24,006
2008	32,479
2009	36,537
2010	65,255
2011	108,249
2012	156,508
2013	170,910

Spreadsheet at LarsonPrecalculus.com

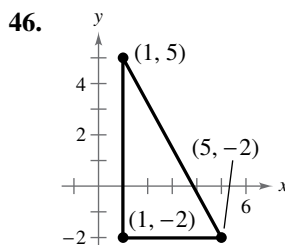
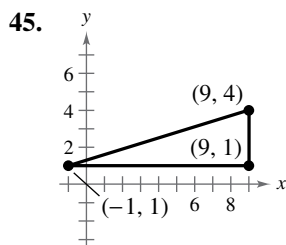
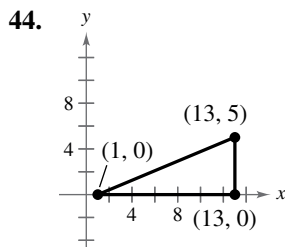
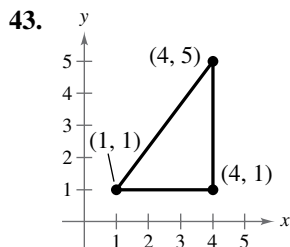
- 32. Meteorology** The table shows the lowest temperature on record  $y$  (in degrees Fahrenheit) in Flagstaff, Arizona, for each month  $x$ , where  $x = 1$  represents January. Sketch a scatter plot of the data. (Source: National Oceanic and Atmospheric Administration)

Month, $x$	Temperature, $y$
1	-22
2	-23
3	-16
4	-2
5	14
6	22
7	32
8	24
9	23
10	-2
11	-13
12	-23

**Finding a Distance** In Exercises 33–42, find the distance between the points algebraically and confirm graphically by using centimeter graph paper and a centimeter ruler.

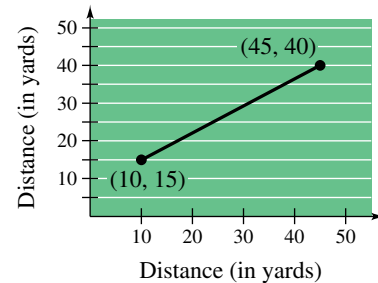
33.  $(6, -3), (6, 5)$       34.  $(-11, 4), (-1, 4)$   
 35.  $(-2, 6), (3, -6)$       36.  $(8, 5), (0, 20)$   
 37.  $(2, 6), (-5, 5)$   
 38.  $(-3, -7), (1, -15)$   
 39.  $(\frac{1}{2}, \frac{4}{3}), (2, -1)$   
 40.  $(-\frac{2}{3}, 3), (-1, \frac{5}{4})$   
 41.  $(-4.2, 3.1), (-12.5, 4.8)$   
 42.  $(9.5, -2.6), (-3.9, 8.2)$

**Verifying a Right Triangle** In Exercises 43–46, (a) find the length of each side of the right triangle and (b) show that these lengths satisfy the Pythagorean Theorem.

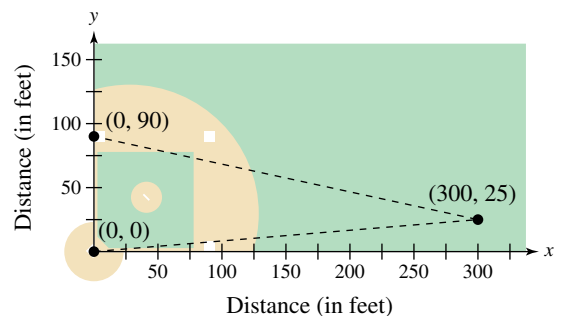


**Verifying a Polygon** In Exercises 47–54, show that the points form the vertices of the polygon.

47. Right triangle:  $(4, 0), (2, 1), (-1, -5)$   
 48. Right triangle:  $(-1, 3), (3, 5), (5, 1)$   
 49. Isosceles triangle:  $(1, -3), (3, 2), (-2, 4)$   
 50. Isosceles triangle:  $(2, 3), (4, 9), (-2, 7)$   
 51. Parallelogram:  $(2, 5), (0, 9), (-2, 0), (0, -4)$   
 52. Parallelogram:  $(0, 1), (3, 7), (4, 4), (1, -2)$   
 53. Rectangle:  $(-5, 6), (0, 8), (-3, 1), (2, 3)$  (Hint: Show that the diagonals are of equal length.)  
 54. Rectangle:  $(2, 4), (3, 1), (1, 2), (4, 3)$  (Hint: Show that the diagonals are of equal length.)  
 55. **Physical Education** In a football game, a quarterback throws a pass from the 15-yard line, 10 yards from the sideline, as shown in the figure. The pass is caught on the 40-yard line, 45 yards from the same sideline. How long is the pass?



56. **Physical Education** A major league baseball diamond is a square with 90-foot sides. In the figure, home plate is at the origin and the first base line lies on the positive  $x$ -axis. The right fielder fields the ball at the point  $(300, 25)$ . How far does the right fielder have to throw the ball to get a runner out at home plate? How far does the right fielder have to throw the ball to get a runner out at third base? (Round your answers to one decimal place.)



57. **Aviation** A jet plane flies from Naples, Italy, in a straight line to Rome, Italy, which is 120 kilometers north and 150 kilometers west of Naples. How far does the plane fly?

**58. Geography** Oklahoma City, Oklahoma, is about 514 miles east of Albuquerque, New Mexico, and about 360 miles north of Austin, Texas. Find the approximate distance between Albuquerque and Austin.

**Finding a Line Segment's Midpoint** In Exercises 59–68, (a) plot the points and (b) find the midpoint of the line segment joining the points.

- 59. (0, 0), (8, 6)
- 60. (1, 12), (9, 0)
- 61. (-1, 2), (5, 4)
- 62. (2, 10), (10, 2)
- 63. (-4, 10), (4, -5)
- 64. (-7, -4), (2, 8)
- 65.  $(\frac{1}{2}, 1), (-\frac{5}{2}, \frac{4}{3})$
- 66.  $(-\frac{1}{3}, -\frac{1}{3}), (-\frac{1}{6}, -\frac{1}{2})$
- 67. (6.2, 5.4), (-3.7, 1.8)
- 68. (-16.8, 12.3), (5.6, 4.9)

**Estimating Annual Revenues** In Exercises 69 and 70, use the Midpoint Formula to estimate the annual revenues (in millions of dollars) for Texas Roadhouse and Papa John's in 2011. The revenues for the two companies in 2009 and 2013 are shown in the tables. Assume that the revenues followed a linear pattern. (Sources: Texas Roadhouse, Inc.; Papa John's International)

69. Texas Roadhouse

Year	Annual revenue (in millions of dollars)
2009	942
2013	1423

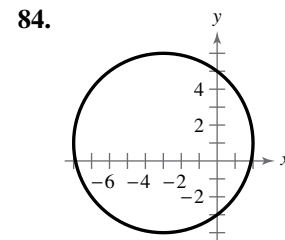
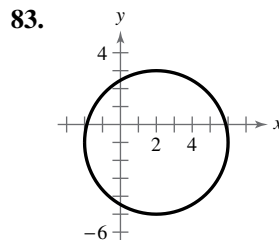
70. Papa John's Intl.

Year	Annual revenue (in millions of dollars)
2009	1106
2013	1439

**Writing an Equation of a Circle** In Exercises 71–84, write the standard form of the equation of the specified circle.

- 71. Center: (0, 0); radius: 5
- 72. Center: (0, 0); radius: 6
- 73. Center: (2, -1); radius: 4
- 74. Center: (-5, 3); radius: 2
- 75. Center: (-1, 2); solution point: (0, 0)

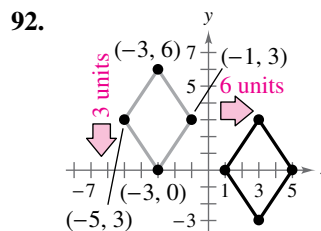
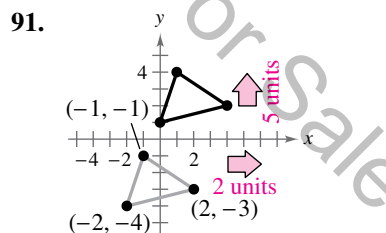
- 76. Center: (3, -2); solution point: (-1, 1)
- 77. Endpoints of a diameter: (-4, -1), (4, 1)
- 78. Endpoints of a diameter: (0, 0), (6, 8)
- 79. Center: (-2, 1); tangent to the x-axis
- 80. Center: (3, -2); tangent to the y-axis
- 81. The circle inscribed in the square with vertices (7, -2), (-1, -2), (-1, -10), and (7, -10)
- 82. The circle inscribed in the square with vertices (-12, 10), (8, 10), (8, -10), and (-12, -10)



**Sketching a Circle** In Exercises 85–90, find the center and radius, and sketch the circle.

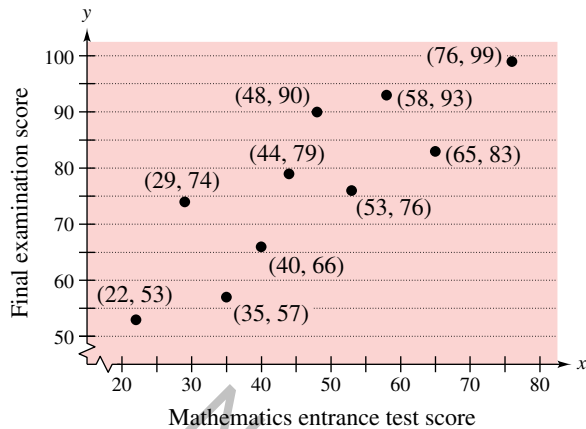
- 85.  $x^2 + y^2 = 25$
- 86.  $x^2 + y^2 = 64$
- 87.  $(x - 6)^2 + y^2 = 9$
- 88.  $(x + 1)^2 + (y - 3)^2 = 4$
- 89.  $(x - \frac{1}{3})^2 + (y + \frac{2}{3})^2 = \frac{16}{9}$
- 90.  $(x + 4.5)^2 + (y - 0.5)^2 = 2.25$

**Translating Points in the Plane** In Exercises 91–94, the polygon is shifted to a new position in the plane. Find the coordinates of the vertices of the polygon in the new position.

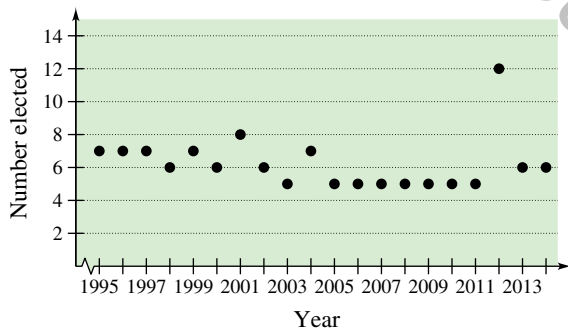


- 93. Original coordinates of vertices: (0, 2), (-3, 5), (-5, 2), (-2, -1)  
Shift: three units upward, one unit to the left
- 94. Original coordinates of vertices: (1, -1), (3, 2), (1, -2)  
Shift: two units downward, three units to the left

95. **Education** The scatter plot shows the mathematics entrance test scores  $x$  and the final examination scores  $y$  in an algebra course for a sample of 10 students.



- (a) Find the entrance exam score of any student with a final exam score in the 80s.  
 (b) Does a higher entrance exam score necessarily imply a higher final exam score? Explain.
96. **Why you should learn it** (p. B1) The graph shows the numbers of performers who were elected to the Rock and Roll Hall of Fame from 1995 through 2014. Describe any trends or irregularities in the data. From these trends, predict the number of performers elected in 2015. (Source: rockhall.com)



**Conclusions**

**True or False?** In Exercises 97 and 98, determine whether the statement is true or false. Justify your answer.

97. The points  $(-8, 4)$ ,  $(2, 11)$ , and  $(-5, 1)$  represent the vertices of an isosceles triangle.  
 98. If four points represent the vertices of a polygon, and the four sides are equal, then the polygon must be a square.  
 99. **Think About It** What is the  $y$ -coordinate of any point on the  $x$ -axis? What is the  $x$ -coordinate of any point on the  $y$ -axis?

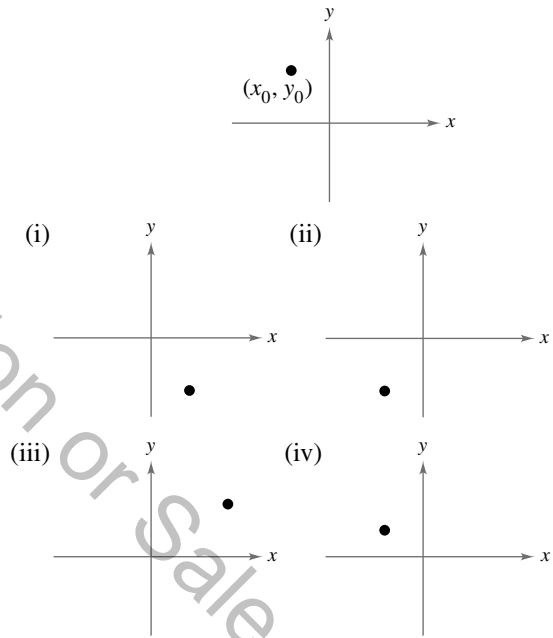
100. **Exploration** A line segment has  $(x_1, y_1)$  as one endpoint and  $(x_m, y_m)$  as its midpoint. Find the other endpoint  $(x_2, y_2)$  of the line segment in terms of  $x_1, y_1, x_m,$  and  $y_m$ . Use the result to find the coordinates of the endpoint of a line segment when the coordinates of the other endpoint and midpoint are, respectively,

- (a)  $(1, -2), (4, -1)$ .  
 (b)  $(-5, 11), (2, 4)$ .

101. **Exploration** Use the Midpoint Formula three times to find the three points that divide the line segment joining  $(x_1, y_1)$  and  $(x_2, y_2)$  into four parts. Use the result to find the points that divide the line segment joining the given points into four equal parts.

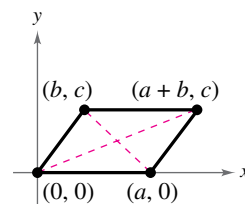
- (a)  $(1, -2), (4, -1)$       (b)  $(-2, -3), (0, 0)$

102. **Think About It** Use the plot of the point  $(x_0, y_0)$  in the figure. Match the transformation of the point with the correct plot. Explain your reasoning. [The plots are labeled (i), (ii), (iii), and (iv).]



- (a)  $(x_0, -y_0)$       (b)  $(-2x_0, y_0)$   
 (c)  $(x_0, \frac{1}{2}y_0)$       (d)  $(-x_0, -y_0)$

103. **Proof** Prove that the diagonals of the parallelogram in the figure intersect at their midpoints.



## B.2 Graphs of Equations

### The Graph of an Equation

News magazines often show graphs comparing the rate of inflation, the federal deficit, or the unemployment rate to the time of year. Businesses use graphs to report monthly sales statistics. Such graphs provide geometric pictures of the way one quantity changes with respect to another. Frequently, the relationship between two quantities is expressed as an **equation**. This appendix introduces the basic procedure for determining the geometric picture associated with an equation.

For an equation in the variables  $x$  and  $y$ , a point  $(a, b)$  is a **solution point** when substitution of  $a$  for  $x$  and  $b$  for  $y$  satisfies the equation. Most equations have *infinitely many* solution points. For example, the equation

$$3x + y = 5$$

has solution points

$$(0, 5), (1, 2), (2, -1), (3, -4)$$

and so on. The set of all solution points of an equation is the **graph of the equation**.

#### What you should learn

- ▶ Sketch graphs of equations by point plotting.
- ▶ Graph equations using a graphing utility.
- ▶ Use graphs of equations to solve real-life problems.

#### Why you should learn it

The graph of an equation can help you see relationships between real-life quantities. For example, in Exercise 64 on page B20, a graph can be used to analyze the life expectancies of children born in the United States.

#### EXAMPLE 1 Determining Solution Points

Determine whether (a)  $(2, 13)$  and (b)  $(-1, -3)$  lie on the graph of

$$y = 10x - 7.$$

#### Solution

a.  $y = 10x - 7$       Write original equation.  
 $13 \stackrel{?}{=} 10(2) - 7$       Substitute 2 for  $x$  and 13 for  $y$ .  
 $13 = 13$        $(2, 13)$  is a solution. ✓

The point  $(2, 13)$  *does* lie on the graph of  $y = 10x - 7$  because it is a solution point of the equation.

b.  $y = 10x - 7$       Write original equation.  
 $-3 \stackrel{?}{=} 10(-1) - 7$       Substitute  $-1$  for  $x$  and  $-3$  for  $y$ .  
 $-3 \neq -17$        $(-1, -3)$  is not a solution.

The point  $(-1, -3)$  *does not* lie on the graph of  $y = 10x - 7$  because it is not a solution point of the equation. ■

The basic technique used for sketching the graph of an equation is the point-plotting method.

#### Sketching the Graph of an Equation by Point Plotting

1. If possible, rewrite the equation so that one of the variables is isolated on one side of the equation.
2. Make a table of values showing several solution points.
3. Plot these points in a rectangular coordinate system.
4. Connect the points with a smooth curve or line.



**EXAMPLE 2** Sketching a Graph by Point Plotting

Use point plotting and graph paper to sketch the graph of  $3x + y = 6$ .

**Solution**

In this case, you can isolate the variable  $y$ .

$$y = 6 - 3x \quad \text{Solve equation for } y.$$

Using negative and positive values of  $x$ , and  $x = 0$ , you can obtain the following table of values (solution points).

$x$	-1	0	1	2	3
$y = 6 - 3x$	9	6	3	0	-3
Solution point	(-1, 9)	(0, 6)	(1, 3)	(2, 0)	(3, -3)

Next, plot the solution points and connect them, as shown in Figure B.13. It appears that the points lie on a line. You will study lines extensively in Section 1.1.

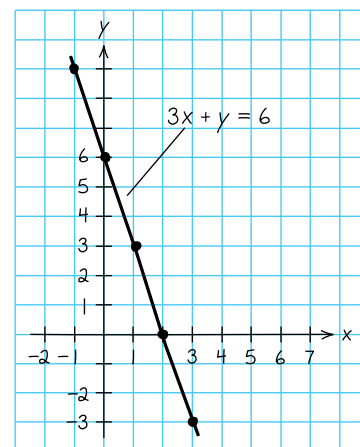


Figure B.13

The points at which a graph touches or crosses an axis are called the **intercepts** of the graph. For instance, in Example 2 the point (0, 6) is the  $y$ -intercept of the graph because the graph crosses the  $y$ -axis at that point. The point (2, 0) is the  $x$ -intercept of the graph because the graph crosses the  $x$ -axis at that point.

**EXAMPLE 3** Sketching a Graph by Point Plotting

See *LarsonPrecalculus.com* for an interactive version of this type of example.

Use point plotting and graph paper to sketch the graph of

$$y = x^2 - 2.$$

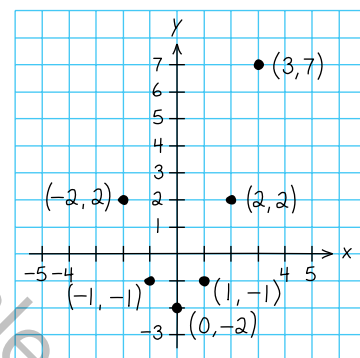
**Solution**

Because the equation is already solved for  $y$ , make a table of values by choosing several convenient values of  $x$  and calculating the corresponding values of  $y$ .

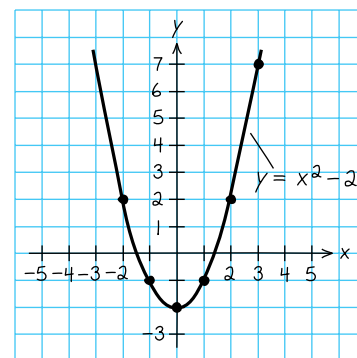
$x$	-2	-1	0	1	2	3
$y = x^2 - 2$	2	-1	-2	-1	2	7
Solution point	(-2, 2)	(-1, -1)	(0, -2)	(1, -1)	(2, 2)	(3, 7)

Next, plot the solution points, as shown in Figure B.14(a). Finally, connect the points with a smooth curve, as shown in Figure B.14(b). This graph is called a *parabola*. You will study parabolas in Section 2.1.

In this text, you will study two basic ways to create graphs: *by hand* and *using a graphing utility*. For instance, the graphs in Figures B.13 and B.14 were sketched by hand, and the graph in Figure B.17 (on the next page) was created using a graphing utility.



(a)



(b)

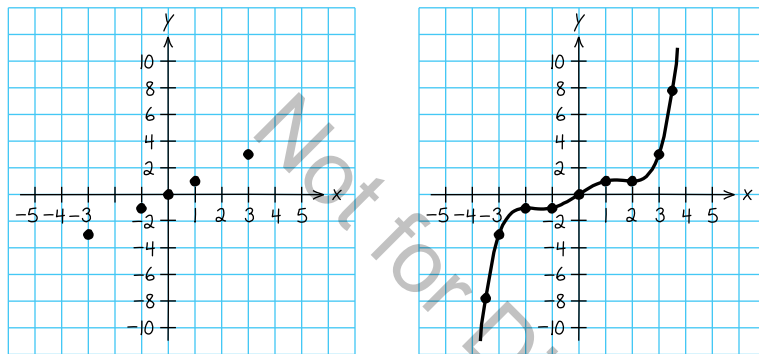
Figure B.14

## Using a Graphing Utility

One of the disadvantages of the point-plotting method is that to get a good idea about the shape of a graph, you need to plot *many* points. With only a few points, you could misrepresent the graph of an equation. For instance, consider the equation

$$y = \frac{1}{30}x(x^4 - 10x^2 + 39).$$

When you plot the points  $(-3, -3)$ ,  $(-1, -1)$ ,  $(0, 0)$ ,  $(1, 1)$ , and  $(3, 3)$ , as shown in Figure B.15(a), you might think that the graph of the equation is a line. This is not correct. By plotting several more points and connecting the points with a smooth curve, you can see that the actual graph is not a line, as shown in Figure B.15(b).



(a)  
Figure B.15

(b)

From this, you can see that the point-plotting method leaves you with a dilemma. This method can be very inaccurate when only a few points are plotted, and it is very time-consuming to plot a dozen (or more) points. Technology can help solve this dilemma. Plotting several (even several hundred) points in a rectangular coordinate system is something that a computer or calculator can do easily. For instance, you can enter the equation  $y = \frac{1}{30}x(x^4 - 10x^2 + 39)$  in a graphing utility (see Figure B.16) to obtain the graph shown in Figure B.17.

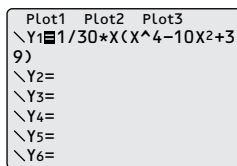


Figure B.16

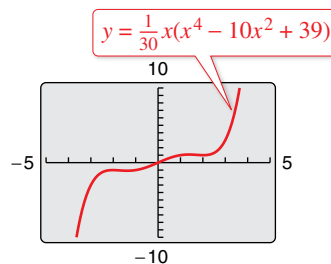


Figure B.17

## Technology Tip

Many graphing utilities are capable of creating a table of values such as the following, which shows some points of the graph in Figure B.15(b). For instructions on how to use the *table* feature, see Appendix A; for specific keystrokes, go to this textbook's *Companion Website*.

X	Y1
-2	-1
-1	-1
0	0
1	1
2	1
3	3
4	18

X=-2

## Technology Tip

By choosing different viewing windows for a graph, it is possible to obtain very different impressions of the graph's shape. For instance, Figure B.18 shows a different viewing window for the graph of the equation in Figure B.17. Note how Figure B.18 does not show *all* of the important features of the graph as does Figure B.17. For instructions on how to set up a viewing window, see Appendix A; for specific keystrokes, go to this textbook's *Companion Website*.

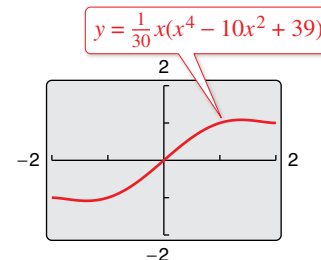


Figure B.18

### Using a Graphing Utility to Graph an Equation

To graph an equation involving  $x$  and  $y$  on a graphing utility, do the following.

1. Rewrite the equation so that  $y$  is isolated on the left side.
2. Enter the equation in the graphing utility.
3. Determine a *viewing window* that shows all important features of the graph.
4. Graph the equation.

**EXAMPLE 4** Using a Graphing Utility to Graph an Equation

To graph

$$y = -0.5x^3 + 2x$$

enter the equation in a graphing utility. Then use a standard viewing window (see Figure B.19) to obtain the graph shown in Figure B.20.

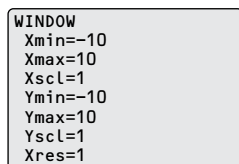


Figure B.19

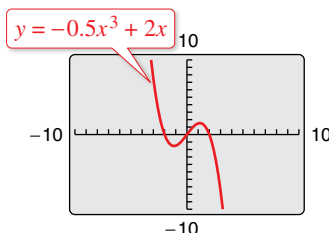


Figure B.20

**EXAMPLE 5** Using a Graphing Utility to Graph a Circle

Use a graphing utility to graph  $x^2 + y^2 = 9$ .

**Solution**

The graph of  $x^2 + y^2 = 9$  is a circle whose center is the origin and whose radius is 3. To graph the equation, begin by solving the equation for  $y$ .

$$\begin{aligned} x^2 + y^2 &= 9 && \text{Write original equation.} \\ y^2 &= 9 - x^2 && \text{Subtract } x^2 \text{ from each side.} \\ y &= \pm \sqrt{9 - x^2} && \text{Take the square root of each side.} \end{aligned}$$

Remember that when you take the square root of a variable expression, you must account for both the positive and negative solutions. The graph of  $y = \sqrt{9 - x^2}$  is the upper semicircle. The graph of  $y = -\sqrt{9 - x^2}$  is the lower semicircle. Enter *both* equations in your graphing utility and generate the resulting graphs. In Figure B.21, note that for a standard viewing window, the two graphs do not appear to form a circle. You can overcome this problem by using a *square setting*, in which the horizontal and vertical tick marks have equal spacing, as shown in Figure B.22. On many graphing utilities, a square setting can be obtained when the ratio of  $y$  to  $x$  is 2 to 3. For instance, in Figure B.22, the ratio of  $y$  to  $x$  is

$$\frac{Y_{\max} - Y_{\min}}{X_{\max} - X_{\min}} = \frac{4 - (-4)}{6 - (-6)} = \frac{8}{12} = \frac{2}{3}$$

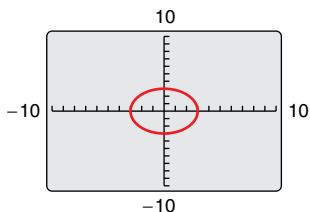


Figure B.21

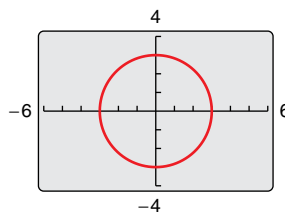


Figure B.22

**Technology Tip**

The standard viewing window on many graphing utilities does not give a true geometric perspective because the screen is rectangular, which distorts the image. That is, perpendicular lines will not appear to be perpendicular, and circles will not appear to be circular. To overcome this, you can use a *square setting*, as demonstrated in Example 5.

## Applications

Throughout this course, you will learn that there are many ways to approach a problem. Three common approaches are illustrated in Examples 6 and 7.

*An Algebraic Approach:* Use the rules of algebra.

*A Graphical Approach:* Draw and use a graph.

*A Numerical Approach:* Construct and use a table.

You should develop the habit of using at least two approaches to solve every problem in order to build your intuition and to check that your answer is reasonable.

The following two applications show how to develop mathematical models to represent real-world situations. You will see that both a graphing utility and algebra can be used to understand and solve the problems posed.

### EXAMPLE 6 Running a Marathon

A runner runs at a constant rate of 4.8 miles per hour. The verbal model and algebraic equation relating distance run and elapsed time are as follows.

*Verbal Model:* Distance = Rate • Time

*Equation:*  $d = 4.8t$

- Determine how far the runner can run in 3.1 hours.
- Determine how long it will take the runner to run a 26.2-mile marathon.

#### Algebraic Solution

- To begin, find how far the runner can run in 3.1 hours by substituting 3.1 for  $t$  in the equation.

$$\begin{aligned} d &= 4.8t && \text{Write original equation.} \\ &= 4.8(3.1) && \text{Substitute 3.1 for } t. \\ &\approx 14.9 && \text{Use a calculator.} \end{aligned}$$

So, the runner can run about 14.9 miles in 3.1 hours. Use estimation to check your answer. Because 4.8 is about 5 and 3.1 is about 3, the distance is about  $5(3) = 15$ . So, 14.9 is reasonable.

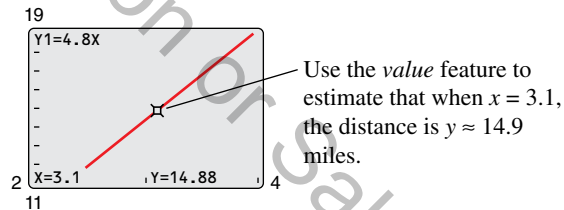
- You can find how long it will take to run a 26.2-mile marathon as follows. (For help with solving linear equations, see Appendix E at this textbook's *Companion Website*.)

$$\begin{aligned} d &= 4.8t && \text{Write original equation.} \\ 26.2 &= 4.8t && \text{Substitute 26.2 for } d. \\ \frac{26.2}{4.8} &= t && \text{Divide each side by 4.8.} \\ 5.5 &\approx t && \text{Use a calculator.} \end{aligned}$$

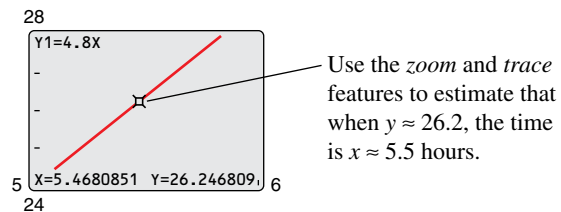
So, it will take the runner about 5.5 hours to run 26.2 miles.

#### Graphical Solution

- Use a graphing utility to graph  $d = 4.8t$ . (Represent  $d$  by  $y$  and  $t$  by  $x$ .) Choose a viewing window that shows the graph at  $x = 3.1$ .



- Adjust the viewing window so that it shows the graph at  $y = 26.2$ .



### EXAMPLE 7 Monthly Wage

You receive a monthly salary of \$2000 plus a commission of 10% of sales. The verbal model and algebraic equation relating wages, salary, and commission are as follows.

Verbal Model: Wages = Salary + Commission on sales

Equation:  $y = 2000 + 0.1x$

- Sales are \$1480 in August. What are your wages for that month?
- You receive \$2225 for September. What are your sales for that month?

#### Numerical Solution

- Enter  $y = 2000 + 0.1x$  in a graphing utility. Then use the *table* feature of the graphing utility to create a table. Start the table at  $x = 1400$  with a table step of 10.

X	Y1
1420	2142
1430	2143
1440	2144
1450	2145
1460	2146
1470	2147
1480	2148

When  $x = 1480$ , the wages are  $y = \$2148$ .

- Start the table at  $x = 2000$  with a table step of 100.

X	Y1
2000	2200
2100	2210
2200	2220
2300	2230
2400	2240
2500	2250
2600	2260

From the table, you can see that wages of \$2225 result from sales between \$2200 and \$2300.

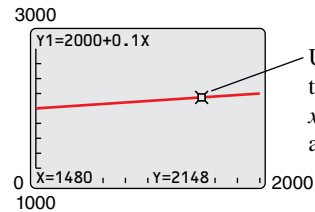
You can improve the estimate by starting the table at  $x = 2200$  with a table step of 10.

From the table, you can see that wages of \$2225 result from sales of \$2250.

X	Y1
2200	2220
2210	2221
2220	2222
2230	2223
2240	2224
2250	2225
2260	2226

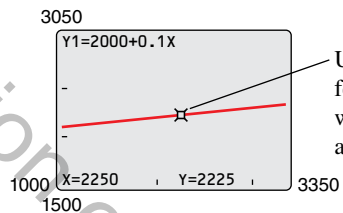
#### Graphical Solution

- Use a graphing utility to graph  $y = 2000 + 0.1x$ . Choose a viewing window that shows the graph at  $x = 1480$ .



Use the *value* feature to estimate that when  $x = 1480$ , the wages are  $y = \$2148$ .

- Use the graphing utility to find the value along the  $x$ -axis (sales) that corresponds to a  $y$ -value of 2225 (wages). Adjust the viewing window so that it shows the graph at  $y = 2225$ .



Use the *zoom* and *trace* features to estimate that when  $y = 2225$ , the sales are  $x = \$2250$ .

Remember to use a different approach to check that your answer is reasonable. For instance, to check the numerical solution to Example 7, use a graphical approach as shown above or use an algebraic approach as follows.

- Substitute 1480 for  $x$  in the original equation and solve for  $y$ .

$$y = 2000 + 0.1(1480) = \$2148$$

- Substitute 2225 for  $y$  in the original equation and solve for  $x$ .

$$2225 = 2000 + 0.1x \quad \rightarrow \quad x = \$2250$$

## B.2 Exercises

See *CalcChat.com* for tutorial help and worked-out solutions to odd-numbered exercises. For instructions on how to use a graphing utility, see Appendix A.

### Vocabulary and Concept Check

In Exercises 1 and 2, fill in the blank.

- For an equation in  $x$  and  $y$ , if substitution of  $a$  for  $x$  and  $b$  for  $y$  satisfies the equation, then the point  $(a, b)$  is a \_\_\_\_\_.
- The set of all solution points of an equation is the \_\_\_\_\_ of the equation.
- Name three common approaches you can use to solve problems mathematically.
- List the steps for sketching the graph of an equation by point plotting.

### Procedures and Problem Solving

**Determining Solution Points** In Exercises 5–12, determine whether each point lies on the graph of the equation.

Equation	Points	
5. $y = \sqrt{x + 4}$	(a) (0, 2)	(b) (12, 4)
6. $y = \sqrt{5 - x}$	(a) (1, 2)	(b) (5, 0)
7. $y = 4 -  x - 2 $	(a) (1, 5)	(b) (1.2, 3.2)
8. $y =  x - 1  + 2$	(a) (2, 1)	(b) (3.2, 4.2)
9. $2x - y - 3 = 0$	(a) (1, 2)	(b) (1, -1)
10. $x^2 + y^2 = 20$	(a) (3, -2)	(b) (-4, 2)
11. $y = x^2 - 3x + 2$	(a) $(\frac{5}{2}, \frac{3}{4})$	(b) (-2, 8)
12. $y = \frac{1}{3}x^3 - 2x^2$	(a) $(2, -\frac{16}{3})$	(b) (-3, 9)

**Sketching a Graph by Point Plotting** In Exercises 13–16, complete the table. Use the resulting solution points to sketch the graph of the equation. Use a graphing utility to verify the graph.

13.  $3x - 2y = 2$

$x$	-2	0	$\frac{2}{3}$	1	2
$y$					
Solution point					

14.  $-4x + 2y = 10$

$x$	-3	$-\frac{5}{2}$	0	1	2
$y$					
Solution point					

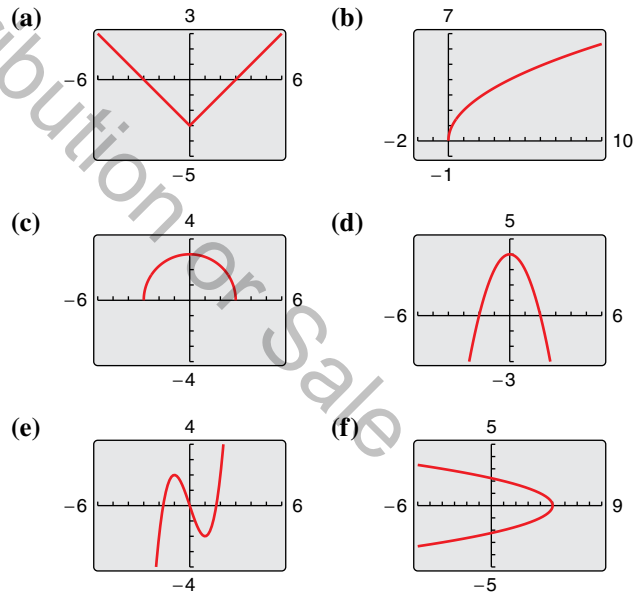
15.  $2x + y = x^2$

$x$	-1	0	1	2	3
$y$					
Solution point					

16.  $6x - 2y = -2x^2$

$x$	-4	-3	-2	0	1
$y$					
Solution point					

**Matching an Equation with Its Graph** In Exercises 17–22, match the equation with its graph. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]



17.  $y = 2\sqrt{x}$

19.  $y = \sqrt{9 - x^2}$

21.  $y = x^3 - 3x$

18.  $y = 4 - x^2$

20.  $y = |x| - 3$

22.  $x = 5 - y^2$

**Sketching the Graph of an Equation** In Exercises 23–30, sketch the graph of the equation.

23.  $y = 2 - x^2$

25.  $y = \sqrt{x - 3}$

27.  $y = |x - 2|$

29.  $x = y^2 - 1$

24.  $y = x^3 - 3$

26.  $y = \sqrt{1 - x}$

28.  $y = 4 - |x|$

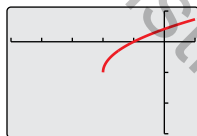
30.  $x = y^2 + 4$

**Using a Graphing Utility to Graph an Equation** In Exercises 31–44, use a graphing utility to graph the equation. Use a standard viewing window. Approximate any  $x$ - or  $y$ -intercepts of the graph.

31.  $y = 5 - \frac{3}{2}x$       32.  $y = \frac{2}{3}x - 1$   
 33.  $y = |x + 2| - 3$       34.  $y = -|x - 3| + 1$   
 35.  $y = \frac{2x}{x - 1}$       36.  $y = \frac{10}{x^2 + 2}$   
 37.  $y = x\sqrt{x + 3}$   
 38.  $y = (6 - x)\sqrt{x}$   
 39.  $y = \sqrt[3]{x - 8}$   
 40.  $y = \sqrt[3]{x + 1}$   
 41.  $x^2 - y = 4x - 3$   
 42.  $2y - x^2 + 8 = 2x$   
 43.  $y - 4x = x^2(x - 4)$   
 44.  $x^3 + y = 1$

**Describing the Viewing Window of a Graphing Utility** In Exercises 45 and 46, describe the viewing window of the graph shown.

45.  $y = -10x + 50$       46.  $y = \sqrt{x + 2} - 1$



**Verifying a Rule of Algebra** In Exercises 47–50, explain how to use a graphing utility to verify that  $y_1 = y_2$ . Identify the rule of algebra that is illustrated.

47.  $y_1 = \frac{1}{4}(x^2 - 8)$       48.  $y_1 = \frac{1}{2}x + (x + 1)$   
 $y_2 = \frac{1}{4}x^2 - 2$        $y_2 = \frac{3}{2}x + 1$   
 49.  $y_1 = \frac{1}{5}[10(x^2 - 1)]$       50.  $y_1 = (x^2 + 3) \cdot \frac{1}{x^2 + 3}$   
 $y_2 = 2(x^2 - 1)$        $y_2 = 1$

**Using a Graphing Utility to Graph an Equation** In Exercises 51–54, use a graphing utility to graph the equation. Use the *trace* feature of the graphing utility to approximate the unknown coordinate of each solution point accurate to two decimal places. (*Hint:* You may need to use the *zoom* feature of the graphing utility to obtain the required accuracy.)

51.  $y = \sqrt{5 - x}$       52.  $y = x^2(x - 3)$   
 (a) (3,  $y$ )      (a) (-1,  $y$ )  
 (b) ( $x$ , 3)      (b) ( $x$ , 6)  
 53.  $y = x^5 - 5x$       54.  $y = |x^2 - 6x + 5|$   
 (a) (-0.5,  $y$ )      (a) (2,  $y$ )  
 (b) ( $x$ , -2)      (b) ( $x$ , 1.5)

**Using a Graphing Utility to Graph a Circle** In Exercises 55–58, solve for  $y$  and use a graphing utility to graph each of the resulting equations in the same viewing window. (Adjust the viewing window so that the circle appears circular.)

55.  $x^2 + y^2 = 16$   
 56.  $x^2 + y^2 = 36$   
 57.  $(x - 1)^2 + (y - 2)^2 = 49$   
 58.  $(x - 3)^2 + (y - 1)^2 = 25$

**Determining Solution Points** In Exercises 59 and 60, determine which point lies on the graph of the circle. (There may be more than one correct answer.)

59.  $(x - 1)^2 + (y - 2)^2 = 25$   
 (a) (1, 3)      (b) (-2, 6)  
 (c) (5, -1)      (d) (0, 2 + 2 $\sqrt{6}$ )  
 60.  $(x + 2)^2 + (y - 3)^2 = 25$   
 (a) (-2, 3)      (b) (0, 0)  
 (c) (1, -1)      (d) (-1, 3 - 2 $\sqrt{6}$ )

### 61. MODELING DATA

A hospital purchases a new magnetic resonance imaging (MRI) machine for \$500,000. The depreciated value (reduced value)  $y$  after  $t$  years is  $y = 500,000 - 47,000t$ , for  $0 \leq t \leq 9$ .

- (a) Use the constraints of the model and a graphing utility to graph the equation using an appropriate viewing window.  
 (b) Use the *value* feature of the graphing utility to determine the value of  $y$  when  $t = 5.8$ . Verify your answer algebraically.  
 (c) Use the *zoom* and *trace* features of the graphing utility to determine the value of  $t$  when  $y = 156,900$ . Verify your answer algebraically.

### 62. MODELING DATA

You buy a personal watercraft for \$8250. The depreciated value  $y$  after  $t$  years is  $y = 8250 - 689t$ , for  $0 \leq t \leq 10$ .

- (a) Use the constraints of the model and a graphing utility to graph the equation using an appropriate viewing window.  
 (b) Use the *zoom* and *trace* features of the graphing utility to determine the value of  $t$  when  $y = 5545.25$ . Verify your answer algebraically.  
 (c) Use the *value* feature of the graphing utility to determine the value of  $y$  when  $t = 5.5$ . Verify your answer algebraically.



**63. MODELING DATA**

The table shows the numbers of new single-family houses  $y$  (in thousands) in the western United States from 2006 through 2013. (Source: U.S. Census Bureau)

DATA	Year	New houses, $y$ (in thousands)
	2006	415
	2007	294
	2008	190
	2009	118
	2010	103
	2011	91
	2012	101
	2013	129

Spreadsheet at LarsonPrecalculus.com

A model that represents the data is given by

$$y = 13.42t^2 - 294.1t + 1692, \quad 6 \leq t \leq 13$$

where  $t$  represents the year, with  $t = 6$  corresponding to 2006.

- Use the model and the *table* feature of a graphing utility to find the numbers of new houses from 2006 through 2013. How well does the model fit the data? Explain.
- Use the graphing utility to graph the data from the table and the model in the same viewing window. How well does the model fit the data? Explain.
- Use the model to estimate the numbers of new houses in 2015 and 2017. Do the values seem reasonable? Explain.
- Use the *zoom* and *trace* features of the graphing utility to determine during which year(s) the number of new houses was approximately 100,000.

**64. Why you should learn it** (p. B12) The table shows the life expectancy  $y$  of a child (at birth) in the United States for each of the selected years from 1940 through 2010. (Source: U.S. National Center for Health Statistics)

DATA	Year	Life expectancy, $y$
	1940	62.9
	1950	68.2
	1960	69.7
	1970	70.8
	1980	73.7
	1990	75.4
	2000	76.8
	2010	78.7

Spreadsheet at LarsonPrecalculus.com

A model that represents the data is given by

$$y = \frac{63.6 + 0.97t}{1 + 0.01t}, \quad 0 \leq t \leq 70$$

where  $t$  is the time in years, with  $t = 0$  corresponding to 1940.

- Use a graphing utility to graph the data from the table above and the model in the same viewing window. How well does the model fit the data? Explain.
- Find the  $y$ -intercept of the graph of the model. What does it represent in the context of the problem?
- Use the *zoom* and *trace* features of the graphing utility to determine the year when the life expectancy was 70.1. Verify your answer algebraically.
- Determine the life expectancy in 1978 both graphically and algebraically.

**Conclusions**

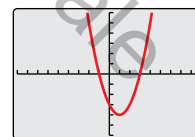
**True or False?** In Exercises 65 and 66, determine whether the statement is true or false. Justify your answer.

- A parabola can have only one  $x$ -intercept.
- The graph of a linear equation can have either no  $x$ -intercepts or only one  $x$ -intercept.
- Writing** Your employer offers you a choice of wage scales: a monthly salary of \$3000 plus commission of 7% of sales or a salary of \$3400 plus a 5% commission. Write a short paragraph discussing how you would choose your option. At what sales level would the options yield the same salary?

**68. Think About It** Use the graph of

$$y = (x - 1)^2 - 4$$

to answer each question.



- Describe the viewing window of the graph.
- Approximate any  $x$ - or  $y$ -intercepts of the graph.
- Determine whether  $(-4, 1)$  and  $(2, -3)$  are solution points of the equation. Explain.



## B.3 Solving Equations Algebraically and Graphically

### Equations and Solutions of Equations

An **equation** in  $x$  is a statement that two algebraic expressions are equal. For example,

$$3x - 5 = 7, \quad x^2 - x - 6 = 0, \quad \text{and} \quad \sqrt{2x} = 4$$

are equations. To **solve** an equation in  $x$  means to find all values of  $x$  for which the equation is true. Such values are **solutions**. For instance,  $x = 4$  is a solution of the equation  $3x - 5 = 7$ , because  $3(4) - 5 = 7$  is a true statement.

The solutions of an equation depend on the kinds of numbers being considered. For instance, in the set of rational numbers,  $x^2 = 10$  has no solution because there is no rational number whose square is 10. In the set of real numbers, however, the equation has two solutions:  $x = \sqrt{10}$  and  $x = -\sqrt{10}$ .

An equation can be classified as an *identity*, a *conditional*, or a *contradiction*, as shown in the following table.

Equation	Definition	Example
<b>Identity</b>	An equation that is true for every real number in the domain of the variable	$\frac{x}{3x^2} = \frac{1}{3x}$ is a true statement for any nonzero real value of $x$ .
<b>Conditional</b>	An equation that is true for some (but not all) of the real numbers in the domain of the variable	$x^2 - 9 = 0$ is a true statement for $x = 3$ and $x = -3$ , but not for any other real values.
<b>Contradiction</b>	An equation that is false for every real number in the domain of the variable	$2x + 1 = 2x - 3$ is a false statement for any real value of $x$ .

A **linear equation in one variable**  $x$  is an equation that can be written in the standard form  $ax + b = 0$ , where  $a$  and  $b$  are real numbers, with  $a \neq 0$ . (For a review of solving one- and two-step linear equations, see Appendix E at this textbook's *Companion Website*.)

To solve an equation involving fractional expressions, find the least common denominator (LCD) of all terms in the equation and multiply every term by this LCD. This procedure clears the equation of fractions.

#### EXAMPLE 1 Solving an Equation Involving Fractions

Solve  $\frac{x}{3} + \frac{3x}{4} = 2$ .

#### Solution

$$\frac{x}{3} + \frac{3x}{4} = 2 \quad \text{Write original equation.}$$

$$(12)\frac{x}{3} + (12)\frac{3x}{4} = (12)2 \quad \text{Multiply each term by the LCD of 12.}$$

$$4x + 9x = 24 \quad \text{Divide out and multiply.}$$

$$13x = 24 \quad \text{Combine like terms.}$$

$$x = \frac{24}{13} \quad \text{Divide each side by 13.}$$

#### What you should learn

- ▶ Solve linear equations.
- ▶ Find  $x$ - and  $y$ -intercepts of graphs of equations.
- ▶ Find solutions of equations graphically.
- ▶ Find the points of intersection of two graphs.
- ▶ Solve polynomial equations.
- ▶ Solve equations involving radicals.
- ▶ Solve equations involving fractions or absolute values.

#### Why you should learn it

Knowing how to solve equations algebraically and graphically can help you solve real-life problems. For instance, in Exercise 195 on page B36, you will find the point of intersection of the graphs of two population models both algebraically and graphically.

After solving an equation, you should check each solution in the original equation. For instance, you can check the solution to Example 1 as follows.

$$\begin{aligned} \frac{x}{3} + \frac{3x}{4} &= 2 && \text{Write original equation.} \\ \frac{\frac{24}{13}}{3} + \frac{3(\frac{24}{13})}{4} &\stackrel{?}{=} 2 && \text{Substitute } \frac{24}{13} \text{ for } x. \\ \frac{8}{13} + \frac{18}{13} &\stackrel{?}{=} 2 && \text{Simplify.} \\ 2 &= 2 && \text{Solution checks. } \checkmark \end{aligned}$$

When multiplying or dividing an equation by a *variable* expression, it is possible to introduce an **extraneous solution**—one that does not satisfy the original equation. The next example demonstrates the importance of checking your solution when you have multiplied or divided by a variable expression.

### EXAMPLE 2 An Equation with an Extraneous Solution

Solve  $\frac{1}{x-2} = \frac{3}{x+2} - \frac{6x}{x^2-4}$ .

#### Solution

The LCD is  $x^2 - 4 = (x + 2)(x - 2)$ . Multiplying each term by the LCD and simplifying produces the following.

$$\begin{aligned} \frac{1}{x-2}(x+2)(x-2) &= \frac{3}{x+2}(x+2)(x-2) - \frac{6x}{x^2-4}(x+2)(x-2) \\ x+2 &= 3(x-2) - 6x, \quad x \neq \pm 2 \\ x+2 &= 3x-6-6x \\ 4x &= -8 \\ x &= -2 && \text{Extraneous solution} \end{aligned}$$

In the original equation,  $x = -2$  yields a denominator of zero. So,  $x = -2$  is an extraneous solution, and the original equation has *no solution*.

#### Remark

Recall that the least common denominator (LCD) of several rational expressions consists of the product of all prime factors in the denominators, with each factor given the highest power of its occurrence in any denominator.

## Intercepts and Solutions

In Appendix B.2, you learned that the intercepts of a graph are the points at which the graph intersects the  $x$ - or  $y$ -axis.

### Definitions of Intercepts

1. The point  $(a, 0)$  is called an  **$x$ -intercept** of the graph of an equation when it is a solution point of the equation. To find the  $x$ -intercept(s), set  $y$  equal to 0 and solve the equation for  $x$ .
2. The point  $(0, b)$  is called a  **$y$ -intercept** of the graph of an equation when it is a solution point of the equation. To find the  $y$ -intercept(s), set  $x$  equal to 0 and solve the equation for  $y$ .

Sometimes it is convenient to denote the  $x$ -intercept as simply the  $x$ -coordinate of the point  $(a, 0)$  rather than the point itself. Likewise, you can denote the  $y$ -intercept as the  $y$ -coordinate of the point  $(0, b)$ . Unless it is necessary to make a distinction, the term *intercept* will refer to either the point or the coordinate.

It is possible for a graph to have no intercepts, one intercept, or several intercepts. For instance, consider the four graphs shown in Figure B.23.

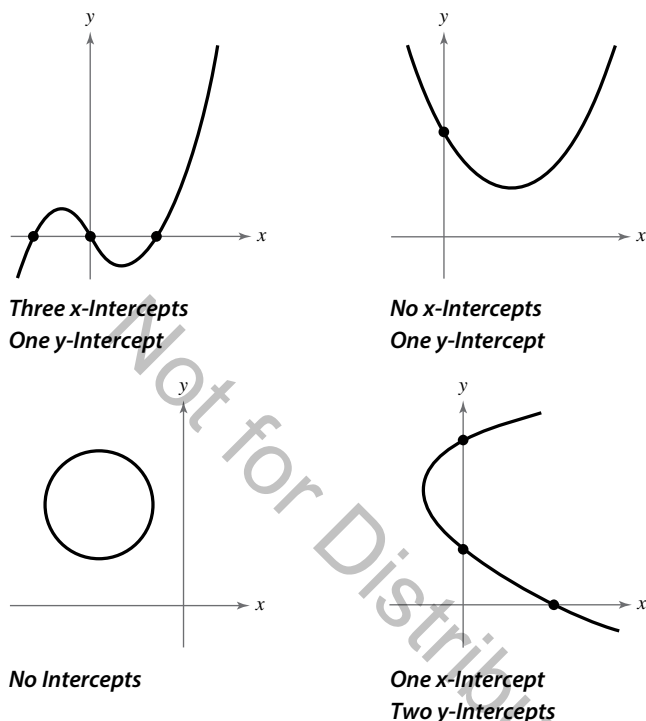


Figure B.23

### EXAMPLE 3 Finding $x$ - and $y$ -Intercepts

Find the  $x$ - and  $y$ -intercepts of the graph of  $2x + 3y = 5$ .

#### Solution

To find the  $x$ -intercept, let  $y = 0$  and solve for  $x$ .

This produces

$$2x = 5 \quad \Rightarrow \quad x = \frac{5}{2}$$

which implies that the graph has one  $x$ -intercept at  $(\frac{5}{2}, 0)$ . To find the  $y$ -intercept, let  $x = 0$  and solve for  $y$ . This produces

$$3y = 5 \quad \Rightarrow \quad y = \frac{5}{3}$$

which implies that the graph has one  $y$ -intercept at  $(0, \frac{5}{3})$ . See Figure B.24.

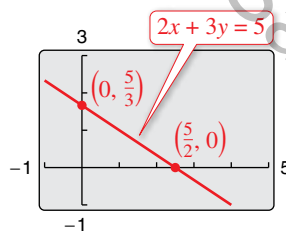


Figure B.24

The concepts of  $x$ -intercepts and solutions of equations are closely related. In fact, the following statements are equivalent when  $a$  is a real number.

1. The point  $(a, 0)$  is an  $x$ -intercept of the graph of  $y = f(x)$ .
2. The number  $a$  is a solution of the equation  $f(x) = 0$ .

The close connection among  $x$ -intercepts and solutions is crucial to your study of algebra. You can take advantage of this connection in two ways. Use your algebraic “equation-solving skills” to find the  $x$ -intercepts of a graph and your “graphing skills” to approximate the solutions of an equation.

## Finding Solutions Graphically

Polynomial equations of degree 1 or 2 can be solved in relatively straightforward ways. Solving polynomial equations of higher degrees can, however, be quite difficult, especially when you rely only on algebraic techniques. For such equations, a graphing utility can be very helpful.

### Graphical Approximations of Solutions of an Equation

1. Write the equation in *general form*,  $f(x) = 0$ , with the nonzero terms on one side of the equation and zero on the other side.
2. Use a graphing utility to graph the function  $y = f(x)$ . Be sure the viewing window shows all the relevant features of the graph.
3. Use the *zero* or *root* feature or the *zoom* and *trace* features of the graphing utility to approximate the  $x$ -intercepts of the graph of  $f$ .

Chapter 2 shows techniques for determining the number of solutions of a polynomial equation. For now, you should know that a polynomial equation of degree  $n$  cannot have more than  $n$  different solutions.

### EXAMPLE 4 Finding Solutions Graphically

Use a graphing utility to approximate any solutions of

$$2x^3 - 3x + 2 = 0.$$

#### Solution

Graph the function  $y = 2x^3 - 3x + 2$ . You can see from the graph shown at the right that there is one  $x$ -intercept. It lies between  $-2$  and  $-1$  and is approximately  $-1.5$ . By using the *zero* or *root* feature of a graphing utility, you can improve the approximation. Choose a left bound of  $x = -2$  (see Figure B.25) and a right bound of  $x = -1$  (see Figure B.26). To two-decimal-place accuracy, the solution is  $x \approx -1.48$ , as shown in Figure B.27. Check this approximation on your calculator. You will find that the value of  $y$  is

$$y = 2(-1.48)^3 - 3(-1.48) + 2 \approx -0.04.$$

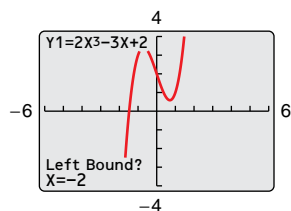
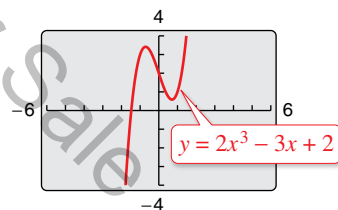


Figure B.25

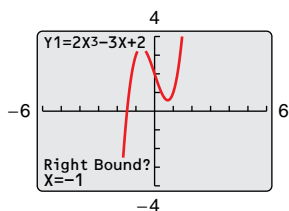


Figure B.26

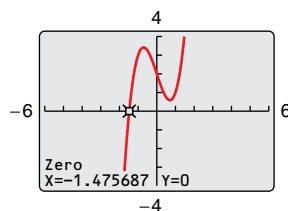


Figure B.27

You can also use a graphing utility's *zoom* and *trace* features to approximate the solution of an equation. Here are some suggestions for using the *zoom-in* feature of a graphing utility.

1. With each successive *zoom-in*, adjust the  $x$ -scale (if necessary) so that the resulting viewing window shows at least one scale mark on each side of the solution.
2. The accuracy of the approximation will always be such that the error is less than the distance between two scale marks. For instance, to approximate the zero to the nearest hundredth, set the  $x$ -scale to 0.01. To approximate the zero to the nearest thousandth, set the  $x$ -scale to 0.001.
3. The graphing utility's *trace* feature can sometimes be used to add one more decimal place of accuracy without changing the viewing window.

Unless stated otherwise, all real solutions in this text will be approximated with an error of *at most* 0.01.

### EXAMPLE 5 Approximating Solutions Graphically

Use a graphing utility to approximate any solutions of

$$x^2 + 3 = 5x.$$

#### Solution

In general form, this equation is

$$x^2 - 5x + 3 = 0. \quad \text{Equation in general form}$$

So, you can begin by graphing

$$y = x^2 - 5x + 3 \quad \text{Function to be graphed}$$

as shown in Figure B.28. This graph has two  $x$ -intercepts, and by using the *zoom* and *trace* features you can approximate the corresponding solutions to be

$$x \approx 0.70 \quad \text{and} \quad x \approx 4.30$$

as shown in Figures B.29 and B.30, respectively.

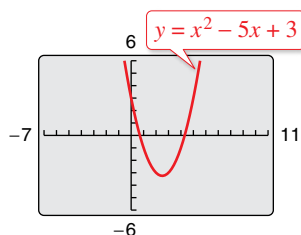


Figure B.28

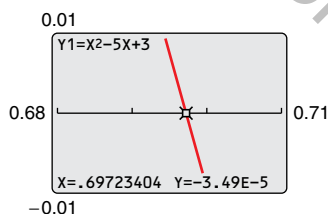


Figure B.29

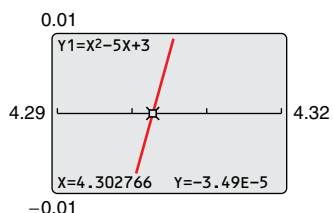


Figure B.30

#### Technology Tip

Use the *zero* or *root* feature of a graphing utility to approximate the solutions of the equation in Example 5 to see that it yields a similar result.

## Points of Intersection of Two Graphs

An ordered pair that is a solution of two different equations is called a **point of intersection** of the graphs of the two equations. For instance, in Figure B.31 you can see that the graphs of the following equations have two points of intersection.

$$y = x + 2 \quad \text{Equation 1}$$

$$y = x^2 - 2x - 2 \quad \text{Equation 2}$$

The point  $(-1, 1)$  is a solution of both equations, and the point  $(4, 6)$  is a solution of both equations. To check this algebraically, substitute

$$x = -1 \quad \text{and} \quad x = 4$$

into each equation.

Check that  $(-1, 1)$  is a solution.

Equation 1:  $y = -1 + 2 = 1$  Solution checks. ✓

Equation 2:  $y = (-1)^2 - 2(-1) - 2 = 1$  Solution checks. ✓

Check that  $(4, 6)$  is a solution.

Equation 1:  $y = 4 + 2 = 6$  Solution checks. ✓

Equation 2:  $y = (4)^2 - 2(4) - 2 = 6$  Solution checks. ✓

To find the points of intersection of the graphs of two equations, solve each equation for  $y$  (or  $x$ ) and set the two results equal to each other. The resulting equation will be an equation in one variable, which can be solved using standard procedures, as shown in Example 6.

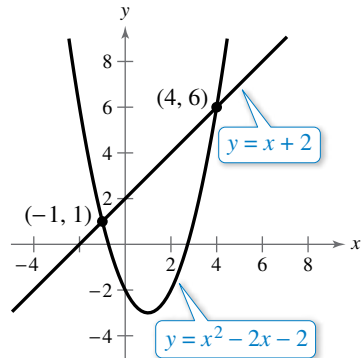


Figure B.31

### EXAMPLE 6 Finding Points of Intersection

Find any points of intersection of the graphs of

$$2x - 3y = -2 \quad \text{and} \quad 4x - y = 6.$$

#### Algebraic Solution

To begin, solve each equation for  $y$  to obtain

$$y = \frac{2}{3}x + \frac{2}{3} \quad \text{and} \quad y = 4x - 6.$$

Next, set the two expressions for  $y$  equal to each other and solve the resulting equation for  $x$ , as follows.

$$\frac{2}{3}x + \frac{2}{3} = 4x - 6 \quad \text{Equate expressions for } y.$$

$$2x + 2 = 12x - 18 \quad \text{Multiply each side by 3.}$$

$$-10x = -20 \quad \text{Subtract } 12x \text{ and } 2 \text{ from each side.}$$

$$x = 2 \quad \text{Divide each side by } -10.$$

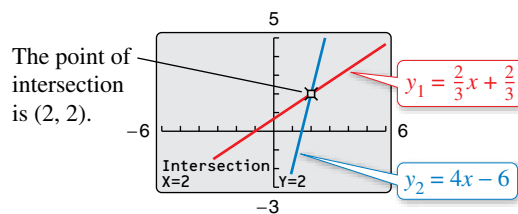
When  $x = 2$ , the  $y$ -value of each of the original equations is 2. So, the point of intersection is  $(2, 2)$ .

#### Graphical Solution

To begin, solve each equation for  $y$  to obtain

$$y_1 = \frac{2}{3}x + \frac{2}{3} \quad \text{and} \quad y_2 = 4x - 6.$$

Then use a graphing utility to graph both equations in the same viewing window, as shown in the figure. Use the *intersect* feature to approximate the point of intersection.



Another way to approximate the points of intersection of two graphs is to graph both equations with a graphing utility and use the *zoom* and *trace* features to find the point or points at which the two graphs intersect.

### EXAMPLE 7 Approximating Points of Intersection Graphically

Use a graphing utility to approximate any points of intersection of the graphs of the following equations.

$$y = x^2 - 3x - 4 \quad \text{Equation 1 (quadratic function)}$$

$$y = x^3 + 3x^2 - 2x - 1 \quad \text{Equation 2 (cubic function)}$$

#### Solution

Begin by using the graphing utility to graph both functions, as shown in Figure B.32. From this display, you can see that the two graphs have only one point of intersection. Then, using the *zoom* and *trace* features, approximate the point of intersection to be

$$(-2.17, 7.25) \quad \text{Point of intersection}$$

as shown in Figure B.33.

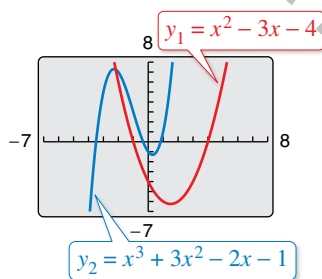


Figure B.32

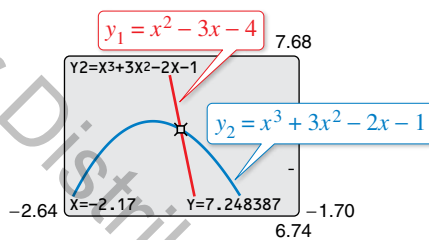


Figure B.33

To test the reasonableness of this approximation, you can evaluate both functions at  $x = -2.17$ .

*Quadratic Function*

$$\begin{aligned} y &= (-2.17)^2 - 3(-2.17) - 4 \\ &\approx 7.22 \end{aligned}$$

*Cubic Function*

$$\begin{aligned} y &= (-2.17)^3 + 3(-2.17)^2 - 2(-2.17) - 1 \\ &\approx 7.25 \end{aligned}$$

Because both functions yield approximately the same  $y$ -value, you can conclude that the approximate coordinates of the point of intersection are

$$x \approx -2.17 \quad \text{and} \quad y \approx 7.25.$$

### Technology Tip

The table shows some points on the graphs of the equations in Example 6. You can find any points of intersection of the graphs by finding all values of  $x$  for which  $y_1$  and  $y_2$  are equal.

X	Y1	Y2
-1	0	-10
0	.66667	-6
1	1.33333	-2
2	2	2
3	2.66667	6
4	3.33333	10
5	4	14
X=2		

### Technology Tip

Use the *intersect* feature of a graphing utility to approximate the point of intersection in Example 7 to see that it yields a similar result.

## Solving Polynomial Equations Algebraically

Polynomial equations can be classified by their *degree*. The degree of a polynomial equation is the highest degree of its terms.

Degree	Name	Example
First	Linear equation	$6x + 2 = 4$
Second	Quadratic equation	$2x^2 - 5x + 3 = 0$
Third	Cubic equation	$x^3 - x = 0$
Fourth	Quartic equation	$x^4 - 3x^2 + 2 = 0$
Fifth	Quintic equation	$x^5 - 12x^2 + 7x + 4 = 0$

You should be familiar with the following four methods for solving quadratic equations *algebraically*.

### Solving a Quadratic Equation

**Factoring:** If  $ab = 0$ , then  $a = 0$  or  $b = 0$ . Zero-Factor Property

Example:  $x^2 - x - 6 = 0$   
 $(x - 3)(x + 2) = 0$   
 $x - 3 = 0$   $\Rightarrow$   $x = 3$   
 $x + 2 = 0$   $\Rightarrow$   $x = -2$

**Extracting Square Roots:** If  $u^2 = c$ , then  $u = \pm\sqrt{c}$ .

Example:  $(x + 3)^2 = 16$   
 $x + 3 = \pm 4$   
 $x = -3 \pm 4$   
 $x = 1$  or  $x = -7$

**Completing the Square:** If  $x^2 + bx = c$ , then

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$$

$$\left(x + \frac{b}{2}\right)^2 = c + \frac{b^2}{4}$$

Example:  $x^2 + 6x = 5$   
 $x^2 + 6x + 3^2 = 5 + 3^2$   
 $(x + 3)^2 = 14$   
 $x + 3 = \pm\sqrt{14}$   
 $x = -3 \pm \sqrt{14}$

**Quadratic Formula:** If  $ax^2 + bx + c = 0$ , then  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ .

Example:  $2x^2 + 3x - 1 = 0$   
 $x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-1)}}{2(2)}$   
 $x = \frac{-3 \pm \sqrt{17}}{4}$

The methods used to solve quadratic equations can sometimes be extended to polynomial equations of higher degree, as shown in the next two examples.



**EXAMPLE 8** Solving a Polynomial Equation by FactoringSolve  $2x^3 - 6x^2 - 6x + 18 = 0$ .**Solution**

This equation has a common factor of 2. You can simplify the equation by first dividing each side of the equation by 2.

$$2x^3 - 6x^2 - 6x + 18 = 0$$

Write original equation.

$$x^3 - 3x^2 - 3x + 9 = 0$$

Divide each side by 2.

$$x^2(x - 3) - 3(x - 3) = 0$$

Factor by grouping.

$$(x - 3)(x^2 - 3) = 0$$

Distributive Property

$$x - 3 = 0 \quad \Rightarrow \quad x = 3$$

Set 1st factor equal to 0.

$$x^2 - 3 = 0 \quad \Rightarrow \quad x = \pm\sqrt{3}$$

Set 2nd factor equal to 0.

The equation has three solutions:

$$x = 3, \quad x = \sqrt{3}, \quad \text{and} \quad x = -\sqrt{3}.$$

Check these solutions in the original equation. Figure B.34 verifies the solutions graphically.

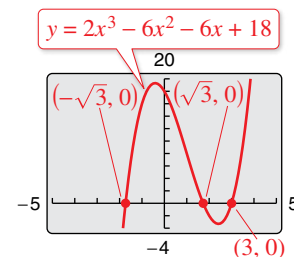


Figure B.34

Occasionally, mathematical models involve equations that are of **quadratic type**. In general, an equation is of quadratic type when it can be written in the form

$$au^2 + bu + c = 0$$

where  $a \neq 0$  and  $u$  is an algebraic expression.

**EXAMPLE 9** Solving an Equation of Quadratic TypeSolve  $x^4 - 3x^2 + 2 = 0$ .**Solution**

This equation is of quadratic type with  $u = x^2$ .

$$x^4 - 3x^2 + 2 = 0$$

Write original equation.

$$(x^2)^2 - 3(x^2) + 2 = 0$$

Write in quadratic form.

$$u^2 - 3u + 2 = 0$$

 $u = x^2$ 

$$(u - 1)(u - 2) = 0$$

Factor.

$$u - 1 = 0 \quad \Rightarrow \quad u = 1$$

Set 1st factor equal to 0.

$$u - 2 = 0 \quad \Rightarrow \quad u = 2$$

Set 2nd factor equal to 0.

Next, replace  $u$  with  $x^2$  and solve for  $x$ .

$$u = 1 \quad \Rightarrow \quad x^2 = 1 \quad \Rightarrow \quad x = \pm 1$$

$$u = 2 \quad \Rightarrow \quad x^2 = 2 \quad \Rightarrow \quad x = \pm\sqrt{2}$$

The equation has four solutions:

$$x = -1, \quad x = 1, \quad x = \sqrt{2}, \quad \text{and} \quad x = -\sqrt{2}.$$

Check these solutions in the original equation. Figure B.35 verifies the solutions graphically.

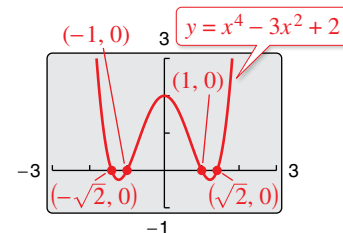


Figure B.35

## Equations Involving Radicals

An equation involving a radical expression can often be cleared of radicals by raising each side of the equation to an appropriate power. When using this procedure, remember to check for extraneous solutions.

### EXAMPLE 10 Solving Equations Involving Radicals

a.  $\sqrt{2x + 7} - x = 2$

$$\sqrt{2x + 7} = x + 2$$

$$2x + 7 = x^2 + 4x + 4$$

$$x^2 + 2x - 3 = 0$$

$$(x + 3)(x - 1) = 0$$

$$x + 3 = 0 \quad \Rightarrow \quad x = -3$$

$$x - 1 = 0 \quad \Rightarrow \quad x = 1$$

By checking these values, you can determine that the only solution is  $x = 1$ . Figure B.36 verifies the solution graphically.

Original equation

Isolate radical.

Square each side.

Write in general form.

Factor.

Set 1st factor equal to 0.

Set 2nd factor equal to 0.

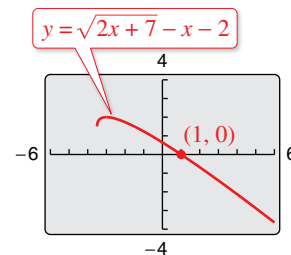


Figure B.36

b.  $\sqrt{2x + 6} - \sqrt{x + 4} = 1$

$$\sqrt{2x + 6} = 1 + \sqrt{x + 4}$$

$$2x + 6 = 1 + 2\sqrt{x + 4} + (x + 4)$$

$$x + 1 = 2\sqrt{x + 4}$$

$$x^2 + 2x + 1 = 4(x + 4)$$

$$x^2 - 2x - 15 = 0$$

$$(x - 5)(x + 3) = 0$$

$$x - 5 = 0 \quad \Rightarrow \quad x = 5$$

$$x + 3 = 0 \quad \Rightarrow \quad x = -3$$

By checking these values, you can determine that the only solution is  $x = 5$ . Figure B.37 verifies the solution graphically.

Original equation

Isolate  $\sqrt{2x + 6}$ .

Square each side.

Isolate  $2\sqrt{x + 4}$ .

Square each side.

Write in general form.

Factor.

Set 1st factor equal to 0.

Set 2nd factor equal to 0.

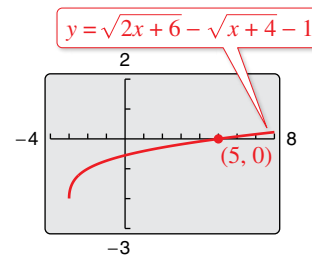


Figure B.37

### EXAMPLE 11 Solving an Equation Involving a Rational Exponent

Solve  $(x + 1)^{2/3} = 4$ .

**Solution**

$$(x + 1)^{2/3} = 4$$

$$\sqrt[3]{(x + 1)^2} = 4$$

$$(x + 1)^2 = 64$$

$$x + 1 = \pm 8$$

$$x = 7, x = -9$$

The solutions are

$$x = 7 \quad \text{and} \quad x = -9.$$

Check these in the original equation. Figure B.38 verifies the solutions graphically.

Write original equation.

Rewrite in radical form.

Cube each side.

Take square root of each side.

Subtract 1 from each side.

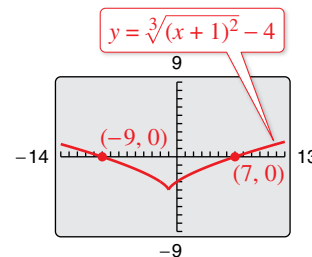


Figure B.38

## Equations Involving Fractions or Absolute Values

As demonstrated in Example 1, you can solve an equation involving fractions algebraically by multiplying each side of the equation by the least common denominator of all terms in the equation to clear the equation of fractions.

### EXAMPLE 12 Solving an Equation Involving Fractions

Solve  $\frac{2}{x} = \frac{3}{x-2} - 1$ .

#### Solution

For this equation, the least common denominator of the three terms is

$$x(x-2)$$

so you can begin by multiplying each term of the equation by this expression.

$$\frac{2}{x} = \frac{3}{x-2} - 1 \quad \text{Write original equation.}$$

$$x(x-2)\frac{2}{x} = x(x-2)\frac{3}{x-2} - x(x-2)(1) \quad \text{Multiply each term by the LCD. (1)}$$

$$2(x-2) = 3x - x(x-2), \quad x \neq 0, 2 \quad \text{Simplify.}$$

$$2x - 4 = -x^2 + 5x \quad \text{Simplify.}$$

$$x^2 - 3x - 4 = 0 \quad \text{Write in general form.}$$

$$(x-4)(x+1) = 0 \quad \text{Factor.}$$

$$x-4 = 0 \quad \Rightarrow \quad x = 4 \quad \text{Set 1st factor equal to 0.}$$

$$x+1 = 0 \quad \Rightarrow \quad x = -1 \quad \text{Set 2nd factor equal to 0.}$$

Check these solutions in the original equation as follows.

Check  $x = 4$

$$\frac{2}{x} = \frac{3}{x-2} - 1$$

$$\frac{2}{4} \stackrel{?}{=} \frac{3}{4-2} - 1$$

$$\frac{1}{2} \stackrel{?}{=} \frac{3}{2} - 1$$

$$\frac{1}{2} = \frac{1}{2} \quad \checkmark$$

Check  $x = -1$

$$\frac{2}{x} = \frac{3}{x-2} - 1$$

$$\frac{2}{-1} \stackrel{?}{=} \frac{3}{-1-2} - 1$$

$$-2 \stackrel{?}{=} -1 - 1$$

$$-2 = -2 \quad \checkmark$$

So, the solutions are

$$x = 4 \quad \text{and} \quad x = -1. \quad \blacksquare$$

**EXAMPLE 13** Solving an Equation Involving an Absolute Value

Solve  $|x^2 - 3x| = -4x + 6$ .

**Solution**

To solve an equation involving an absolute value algebraically, you must consider the fact that the expression inside the absolute value symbols can be positive or negative. This results in *two* separate equations, each of which must be solved.

*First Equation*

$x^2 - 3x = -4x + 6$  Use positive expression.

$x^2 + x - 6 = 0$  Write in general form.

$(x + 3)(x - 2) = 0$  Factor.

$x + 3 = 0$   $\Rightarrow$   $x = -3$  Set 1st factor equal to 0.

$x - 2 = 0$   $\Rightarrow$   $x = 2$  Set 2nd factor equal to 0.

*Second Equation*

$-(x^2 - 3x) = -4x + 6$  Use negative expression.

$x^2 - 7x + 6 = 0$  Write in general form.

$(x - 1)(x - 6) = 0$  Factor.

$x - 1 = 0$   $\Rightarrow$   $x = 1$  Set 1st factor equal to 0.

$x - 6 = 0$   $\Rightarrow$   $x = 6$  Set 2nd factor equal to 0.

**Check**

$|(-3)^2 - 3(-3)| \stackrel{?}{=} -4(-3) + 6$  Substitute -3 for x.  
 $18 = 18$  -3 checks. ✓

$|2^2 - 3(2)| \stackrel{?}{=} -4(2) + 6$  Substitute 2 for x.  
 $2 \neq -2$  2 does not check.

$|1^2 - 3(1)| \stackrel{?}{=} -4(1) + 6$  Substitute 1 for x.  
 $2 = 2$  1 checks. ✓

$|6^2 - 3(6)| \stackrel{?}{=} -4(6) + 6$  Substitute 6 for x.  
 $18 \neq -18$  6 does not check.

The solutions are

$x = -3$  and  $x = 1$ .

Figure B.39 shows the solutions graphically.

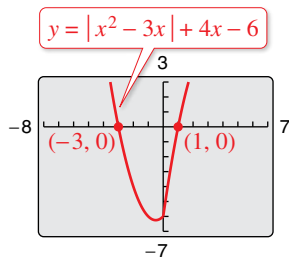


Figure B.39

## B.3 Exercises

See *CalcChat.com* for tutorial help and worked-out solutions to odd-numbered exercises. For instructions on how to use a graphing utility, see Appendix A.

## Vocabulary and Concept Check

In Exercises 1–4, fill in the blank.

1. An \_\_\_\_\_ is a statement that two algebraic expressions are equal.
2. To find all values for which an equation is true is to \_\_\_\_\_ the equation.
3. When solving an equation, it is possible to introduce an \_\_\_\_\_ solution, which is a value that does not satisfy the original equation.
4. An ordered pair that is a solution of two different equations represents a \_\_\_\_\_ of the graphs of the two equations.

## Procedures and Problem Solving

**Checking Solutions** In Exercises 5–10, determine whether each value of  $x$  is a solution of the equation.

Equation	Values	
5. $\frac{5}{2x} - \frac{4}{x} = 3$	(a) $x = -\frac{1}{2}$	(b) $x = 4$
	(c) $x = 0$	(d) $x = \frac{1}{4}$
6. $\frac{x}{2} + \frac{6x}{7} = \frac{19}{14}$	(a) $x = -2$	(b) $x = 1$
	(c) $x = \frac{1}{2}$	(d) $x = 7$
7. $3 + \frac{1}{x+2} = 4$	(a) $x = -1$	(b) $x = -2$
	(c) $x = 0$	(d) $x = 5$
8. $\frac{(x+5)(x-3)}{2} = 24$	(a) $x = -3$	(b) $x = -2$
	(c) $x = 7$	(d) $x = 9$
9. $\frac{\sqrt{x+4}}{6} + 3 = 4$	(a) $x = -3$	(b) $x = 0$
	(c) $x = 21$	(d) $x = 32$
10. $\frac{\sqrt[3]{x-8}}{3} = -\frac{2}{3}$	(a) $x = -16$	(b) $x = 0$
	(c) $x = 9$	(d) $x = 16$

**Classifying an Equation** In Exercises 11–16, determine whether the equation is an identity or a conditional equation.

11.  $2(x-1) = 2x-2$
12.  $-7(x-3) + 4x = 3(7-x)$
13.  $x^2 - 8x + 5 = (x-4)^2 - 11$
14.  $x^2 + 2(3x-2) = x^2 + 6x - 4$
15.  $3 + \frac{1}{x+1} = \frac{4x}{x+1}$
16.  $\frac{5}{x} + \frac{3}{x} = 24$

**Solving an Equation** In Exercises 17–40, solve the equation (if possible). Use a graphing utility to verify your solution.

17.  $3x - 5 = 2x + 7$
18.  $5x + 3 = 6 - 2x$
19.  $4y + 2 - 5y = 7 - 6y$
20.  $5y + 1 = 8y - 5 + 6y$
21.  $3(y - 5) = 3 + 5y$
22.  $5(z - 4) + 4z = 5 - 6z$
23.  $\frac{x}{5} - \frac{x}{2} = 3$
24.  $\frac{5x}{4} + \frac{1}{2} = x - \frac{1}{2}$
25.  $\frac{3}{2}(z + 5) - \frac{1}{4}(z + 24) = 0$
26.  $\frac{3x}{2} + \frac{1}{4}(x - 2) = 10$
27.  $\frac{2(z - 4)}{5} + 5 = 10z$
28.  $\frac{5}{3} + 2(y + 1) = \frac{10}{3}$
29.  $\frac{100 - 4u}{3} = \frac{5u + 6}{4} + 6$
30.  $\frac{17 + y}{y} + \frac{32 + y}{y} = 100$
31.  $\frac{5x - 4}{5x + 4} = \frac{2}{3}$
32.  $\frac{10x + 3}{5x + 6} = \frac{1}{2}$
33.  $\frac{1}{x-3} + \frac{1}{x+3} = \frac{10}{x^2 - 9}$
34.  $\frac{1}{x-2} + \frac{3}{x+3} = \frac{4}{x^2 + x - 6}$

35.  $\frac{7}{2x+1} - \frac{8x}{2x-1} = -4$

36.  $\frac{x}{x+4} + \frac{4}{x+4} + 2 = 0$

37.  $\frac{1}{x} + \frac{2}{x-5} = 0$

38.  $3 = 2 + \frac{2}{z+2}$

39.  $\frac{3}{x^2-3x} + \frac{4}{x} = \frac{1}{x-3}$

40.  $\frac{6}{x} - \frac{2}{x+3} = \frac{3(x+5)}{x(x+3)}$

**Finding x- and y-Intercepts** In Exercises 41–52, find the x- and y-intercepts of the graph of the equation algebraically.

41.  $y = x - 5$

42.  $y = -\frac{3}{4}x - 3$

43.  $y = x^2 + x - 2$

44.  $y = 4 - x^2$

45.  $y = x\sqrt{x+2}$

46.  $y = -\frac{1}{2}x\sqrt{x+3}$

47.  $y = \frac{4}{x}$

48.  $y = \frac{3x-1}{4x}$

49.  $y = |x-2| - 4$

50.  $y = 3 - \frac{1}{2}|x+1|$

51.  $xy - 2y - x + 1 = 0$

52.  $xy - x + 4y = 0$

**Comparing x-Intercepts and Solutions of an Equation** In Exercises 53–56, use a graphing utility to graph the equation and approximate any x-intercepts. Set  $y = 0$  and solve the resulting equation. Compare the results with the x-intercepts of the graph.

53.  $y = 2(x-1) - 4$

54.  $y = 4(x+3) - 2$

55.  $y = 20 - (3x-10)$

56.  $y = 10 + 2(x-2)$

**Checking Solutions** In Exercises 57–62, the solution(s) of the equation are given. Verify the solution(s) both algebraically and graphically.

Equation

Solution(s)

57.  $y = 5(4-x)$

$x = 4$

58.  $y = 3(x-5) + 9$

$x = 2$

59.  $y = x^3 - 6x^2 + 5x$

$x = 0, 5, 1$

60.  $y = x^3 - 9x^2 + 18x$

$x = 0, 3, 6$

61.  $y = \frac{x+2}{3} - \frac{x-1}{5} - 1$

$x = 1$

62.  $y = x - 3 - \frac{10}{x}$

$x = -2, 5$

**Finding Solutions of an Equation Graphically** In Exercises 63–86, write the equation in the form  $y = 0$  (if necessary) and use a graphing utility to approximate any solutions of the equation. Use a calculator to verify your answer.

63.  $2.7x - 0.4x = 1.2$

64.  $3.5x - 8 = 0.5x$

65.  $25(x-3) = 12(x+2) - 10$

66.  $1200 = 300 + 2(x-500)$

67.  $\frac{3x}{2} + \frac{1}{4}(x-2) = 10$

68.  $\frac{2x}{3} + \frac{1}{2}(x-5) = 6$

69.  $x^3 - x + 4 = 0$

70.  $x^3 - 9x^2 + 26x - 27 = 0$

71.  $\frac{2x}{3} = 10 - \frac{24}{x}$

72.  $\frac{x-3}{25} = \frac{x-5}{12}$

73.  $\frac{3}{x+2} - \frac{4}{x-2} = 5$

74.  $\frac{6}{x} + \frac{8}{x+5} = 3$

75.  $(x+2)^2 = x^2 - 6x + 1$

76.  $(x+1)^2 + 2(x-2) = (x+1)(x-2)$

77.  $2x^3 - x^2 - 18x + 9 = 0$

78.  $4x^3 + 12x^2 - 8x - 24 = 0$

79.  $x^4 = 2x^2 - 1$

80.  $5 = 3x^{1/3} + 2x^{2/3}$

81.  $\frac{2}{x+2} = 3$

82.  $\frac{5}{x} = 1 + \frac{3}{x+2}$

83.  $|x-3| = 4$

84.  $|x+1| = 6$

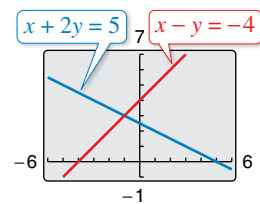
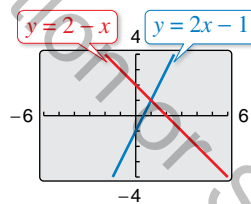
85.  $\sqrt{x-2} = 3$

86.  $\sqrt{x-4} = 8$

**Finding Points of Intersection** In Exercises 87–92, determine any point(s) of intersection of the equations algebraically. Then use a graphing utility to verify your results.

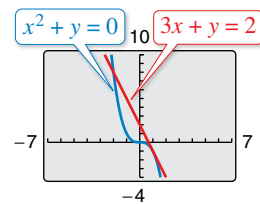
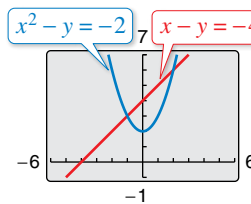
87.  $y = 2 - x$   
 $y = 2x - 1$

88.  $x - y = -4$   
 $x + 2y = 5$



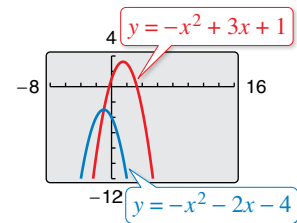
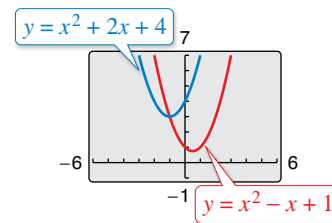
89.  $x - y = -4$   
 $x^2 - y = -2$

90.  $3x + y = 2$   
 $x^3 + y = 0$



91.  $y = x^2 - x + 1$   
 $y = x^2 + 2x + 4$

92.  $y = -x^2 + 3x + 1$   
 $y = -x^2 - 2x - 4$



**Approximating Points of Intersection Graphically** In Exercises 93–98, use a graphing utility to approximate any points of intersection (accurate to three decimal places) of the graphs of the equations. Verify your results algebraically. Test your approximations for reasonableness.

93.  $y = 9 - 2x$

$y = x - 3$

95.  $y = 4 - x^2$

$y = 2x - 1$

97.  $y = 2x^2$

$y = x^4 - 2x^2$

94.  $y = \frac{1}{3}x + 2$

$y = \frac{5}{2}x - 11$

96.  $y = x^3 - 3$

$y = 5 - 2x$

98.  $y = -x$

$y = 2x - x^2$

**Solving a Quadratic Equation by Factoring** In Exercises 99–108, solve the quadratic equation by factoring. Check your solutions in the original equation.

99.  $6x^2 + 3x = 0$

101.  $x^2 - 2x - 8 = 0$

103.  $3 + 5x - 2x^2 = 0$

105.  $x^2 + 4x = 12$

107.  $(x + a)^2 - b^2 = 0$

100.  $9x^2 - 1 = 0$

102.  $x^2 - 10x + 9 = 0$

104.  $2x^2 = 19x + 33$

106.  $-x^2 + 8x = 12$

108.  $x^2 + 2ax + a^2 = 0$

**Solving an Equation by Extracting Square Roots** In Exercises 109–118, solve the equation by extracting square roots. List both the exact solutions and the decimal solutions rounded to two decimal places.

109.  $x^2 = 49$

111.  $(x - 12)^2 = 16$

113.  $(3x - 1)^2 + 6 = 0$

115.  $(2x - 1)^2 = 12$

117.  $(x - 7)^2 = (x + 3)^2$

110.  $x^2 = 144$

112.  $(x - 5)^2 = 25$

114.  $(2x + 3)^2 + 25 = 0$

116.  $(4x + 7)^2 = 44$

118.  $(x + 5)^2 = (x + 4)^2$

**Completing the Square** In Exercises 119–128, solve the quadratic equation by completing the square. Verify your answer graphically.

119.  $x^2 + 4x - 32 = 0$

121.  $x^2 + 6x + 2 = 0$

123.  $9x^2 - 18x + 3 = 0$

125.  $-6 + 2x - x^2 = 0$

127.  $2x^2 + 5x - 8 = 0$

120.  $x^2 - 2x - 3 = 0$

122.  $x^2 + 8x + 14 = 0$

124.  $4x^2 - 4x - 99 = 0$

126.  $-x^2 + x - 1 = 0$

128.  $9x^2 - 12x - 14 = 0$

**Using the Quadratic Formula** In Exercises 129–138, use the Quadratic Formula to solve the equation. Use a graphing utility to verify your solutions graphically.

129.  $2 + 2x - x^2 = 0$

131.  $x^2 + 8x - 4 = 0$

133.  $x^2 + 3x = -8$

135.  $28x - 49x^2 = 4$

130.  $x^2 - 10x + 22 = 0$

132.  $4x^2 - 4x - 4 = 0$

134.  $x^2 + 16 = -5x$

136.  $9x^2 + 24x + 16 = 0$

137.  $3x^2 + 16x + 17 = 0$

138.  $9x^2 - 6x - 37 = 0$

**Choose the Best Method** In Exercises 139–148, solve the equation using any convenient method. Use a graphing utility to verify your solutions graphically.

139.  $x^2 - 2x - 1 = 0$

141.  $(x + 3)^2 = 81$

143.  $x^2 - 14x + 49 = 0$

145.  $x^2 - x - \frac{11}{4} = 0$

147.  $(x + 1)^2 = x^2$

140.  $11x^2 + 33x = 0$

142.  $(x - 1)^2 = -1$

144.  $x^2 - 2x + \frac{13}{4} = 0$

146.  $x^2 + 3x - \frac{3}{4} = 0$

148.  $a^2x^2 - b^2 = 0, a \neq 0$

**Solving Equations Algebraically** In Exercises 149–166, find all solutions of the equation algebraically. Use a graphing utility to verify the solutions graphically.

149.  $4x^4 - 16x^2 = 0$

151.  $5x^3 + 30x^2 + 45x = 0$

152.  $9x^4 - 24x^3 + 16x^2 = 0$

153.  $4x^4 - 18x^2 = 0$

155.  $x^3 - 3x^2 - x + 3 = 0$

156.  $x^4 + 2x^3 - 8x - 16 = 0$

157.  $x^4 - 4x^2 + 3 = 0$

159.  $4x^4 - 65x^2 + 16 = 0$

161.  $\frac{1}{t^2} + \frac{8}{t} + 15 = 0$

163.  $6\left(\frac{s}{s+1}\right)^2 + 5\left(\frac{s}{s+1}\right) - 6 = 0$

164.  $8\left(\frac{t}{t-1}\right)^2 - 2\left(\frac{t}{t-1}\right) - 3 = 0$

165.  $2x + 9\sqrt{x} - 5 = 0$

150.  $8x^4 - 18x^2 = 0$

154.  $20x^3 - 125x = 0$

158.  $x^4 + 5x^2 - 36 = 0$

160.  $36t^4 + 29t^2 - 7 = 0$

162.  $6 - \frac{1}{x} - \frac{1}{x^2} = 0$

166.  $6x - 7\sqrt{x} - 3 = 0$

**Solving Equations Algebraically** In Exercises 167–186, find all solutions of the equation algebraically. Check your solutions both algebraically and graphically.

167.  $\sqrt{x - 10} - 4 = 0$

169.  $\sqrt{x + 1} - 3x = 1$

171.  $\sqrt[3]{2x + 1} + 8 = 0$

173.  $\sqrt{x} - \sqrt{x - 5} = 1$

175.  $(x - 5)^{2/3} = 16$

177.  $3x(x - 1)^{1/2} + 2(x - 1)^{3/2} = 0$

178.  $4x^2(x - 1)^{1/3} + 6x(x - 1)^{4/3} = 0$

179.  $\frac{1}{x} - \frac{1}{x + 1} = 3$

181.  $x = \frac{3}{x} + \frac{1}{2}$

183.  $|2x - 1| = 5$

185.  $|x| = x^2 + x - 3$

168.  $\sqrt{2x + 5} + 3 = 0$

170.  $\sqrt{x + 5} - 2x = 3$

172.  $\sqrt[3]{4x - 3} + 2 = 0$

174.  $\sqrt{x} + \sqrt{x - 20} = 10$

176.  $(x^2 - x - 22)^{4/3} = 16$

180.  $\frac{x}{x^2 - 4} + \frac{1}{x + 2} = 3$

182.  $4x + 1 = \frac{3}{x}$

184.  $|3x + 2| = 7$

186.  $|x - 10| = x^2 - 10x$

**Comparing  $x$ -Intercepts and Solutions of an Equation** In Exercises 187–194, (a) use a graphing utility to graph the equation, (b) use the graph to approximate any  $x$ -intercepts of the graph, (c) set  $y = 0$  and solve the resulting equation, and (d) compare the result of part (c) with the  $x$ -intercepts of the graph.

187.  $y = x^3 - 2x^2 - 3x$       188.  $y = x^4 - 10x^2 + 9$

189.  $y = \sqrt{11x - 30} - x$       190.  $y = 2x - \sqrt{15 - 4x}$

191.  $y = \frac{1}{x} - \frac{4}{x-1} - 1$       192.  $y = x + \frac{9}{x+1} - 5$

193.  $y = |x + 1| - 2$       194.  $y = |x - 2| - 3$

195. **Why you should learn it** (p. B21) The populations (in thousands) of Maryland  $M$  and Wisconsin  $W$  from 2001 through 2013 can be modeled by

$$M = 43.4t + 5355, \quad 1 \leq t \leq 13$$

$$W = 28.4t + 5398, \quad 1 \leq t \leq 13$$

where  $t$  represents the year, with  $t = 1$  corresponding to 2001. (Source: U.S. Census Bureau)

- Use a graphing utility to graph each model in the same viewing window over the appropriate domain. Approximate the point of intersection. Round your result to one decimal place. Explain the meaning of the coordinates of the point.
  - Find the point of intersection algebraically. Round your result to one decimal place. What does the point of intersection represent?
  - Explain the meaning of the slopes of both models and what they tell you about the population growth rates.
  - Use the models to estimate the population of each state in 2016. Do the values seem reasonable? Explain.
196. **Education** The average salaries  $S$  (in thousands of dollars) of secondary classroom teachers in the United States from 2005 through 2013 can be approximated by the model
- $$S = -0.143t^2 + 3.73t + 32.5, \quad 5 \leq t \leq 13$$
- where  $t$  represents the year, with  $t = 5$  corresponding to 2005. (Source: National Education Association)
- Determine algebraically when the average salary of a secondary classroom teacher was \$50,000.
  - Verify your answer to part (a) by creating a table of values for the model.
  - Use a graphing utility to graph the model.
  - Use the model to determine when the average salary reached \$55,500.
  - Do you believe the model could be used to predict the average salaries for years beyond 2013? Explain your reasoning.

197. **Biology** The metabolic rate of an ectothermic organism increases with increasing temperature within a certain range. Experimental data for oxygen consumption  $C$  (in microliters per gram per hour) of a beetle at certain temperatures yielded the model

$$C = 0.45x^2 - 1.65x + 50.75, \quad 10 \leq x \leq 25$$

where  $x$  is the air temperature in degrees Celsius.

- Use a graphing utility to graph the consumption model over the specified domain.
  - Use the graph to approximate the air temperature resulting in oxygen consumption of 150 microliters per gram per hour.
  - The temperature is increased from  $10^\circ\text{C}$  to  $20^\circ\text{C}$ . The oxygen consumption is increased by approximately what factor?
198. **Thermodynamics** The temperature  $T$  (in degrees Fahrenheit) of saturated steam increases as pressure increases. This relationship is approximated by
- $$T = 75.82 - 2.11x + 43.51\sqrt{x}, \quad 5 \leq x \leq 80$$
- where  $x$  is the absolute pressure in pounds per square inch.
- Use a graphing utility to graph the model over the specified domain.
  - The temperature of steam at sea level ( $x = 14.696$ ) is  $212^\circ\text{F}$ . Evaluate the model at this pressure and verify the result graphically.
  - Use the model to approximate the pressure for a steam temperature of  $240^\circ\text{F}$ .

## Conclusions

**True or False?** In Exercises 199 and 200, determine whether the statement is true or false. Justify your answer.

199. Two linear equations can have either one point of intersection or no points of intersection.
200. An equation can never have more than one extraneous solution.
201. **Think About It** Find  $c$  such that  $x = 3$  is a solution to the linear equation  $2x - 5c = 10 + 3c - 3x$ .
202. **Think About It** Find  $c$  such that  $x = 2$  is a solution to the linear equation  $5x + 2c = 12 + 4x - 2c$ .
203. **Exploration** Given that  $a$  and  $b$  are nonzero real numbers, determine the solutions of the equations.
- $ax^2 + bx = 0$
  - $ax^2 - ax = 0$



## B.4 Solving Inequalities Algebraically and Graphically

### Properties of Inequalities

The inequality symbols

$$<, \leq, >, \text{ and } \geq \quad \text{Inequality symbols}$$

are used to compare two numbers and to denote subsets of real numbers. For instance, the simple inequality  $x \geq 3$  denotes all real numbers  $x$  that are greater than or equal to 3.

In this appendix, you will study inequalities that contain more involved statements such as

$$5x - 7 > 3x + 9$$

and

$$-3 \leq 6x - 1 < 3.$$

As with an equation, you **solve an inequality** in the variable  $x$  by finding all values of  $x$  for which the inequality is true. These values are **solutions** of the inequality and are said to **satisfy** the inequality. For instance, the number 9 is a solution of the first inequality listed above because

$$5(9) - 7 > 3(9) + 9$$

$$45 - 7 > 27 + 9$$

$$38 > 36.$$

On the other hand, the number 7 is not a solution because

$$5(7) - 7 \not> 3(7) + 9$$

$$35 - 7 \not> 21 + 9$$

$$28 \not> 30.$$

The set of all real numbers that are solutions of an inequality is the **solution set** of the inequality.

The set of all points on the real number line that represent the solution set is the **graph of the inequality**. Graphs of many types of inequalities consist of intervals on the real number line.

The procedures for solving linear inequalities in one variable are much like those for solving linear equations. To isolate the variable, you can make use of the **properties of inequalities**. These properties are similar to the properties of equality, but there are two important exceptions. When each side of an inequality is multiplied or divided by a negative number, *the direction of the inequality symbol must be reversed* in order to maintain a true statement. Here is an example.

$$-2 < 5$$

Original inequality

$$(-3)(-2) > (-3)(5)$$

Multiply each side by  $-3$  and reverse the inequality symbol.

$$6 > -15$$

Simplify.

Two inequalities that have the same solution set are **equivalent inequalities**. For instance, the inequalities

$$x + 2 < 5 \quad \text{and} \quad x < 3$$

are equivalent. To obtain the second inequality from the first, you can subtract 2 from each side of the inequality. The properties listed at the top of the next page describe operations that can be used to create equivalent inequalities.

#### What you should learn

- ▶ Use properties of inequalities to solve linear inequalities.
- ▶ Solve inequalities involving absolute values.
- ▶ Solve polynomial inequalities.
- ▶ Solve rational inequalities.
- ▶ Use inequalities to model and solve real-life problems.

#### Why you should learn it

An inequality can be used to determine when a real-life quantity exceeds a given level. For instance, Exercises 99–102 on page B49 show how to use linear inequalities to determine when the number of Bed Bath & Beyond stores exceeds the number of Williams-Sonoma stores.

**Properties of Inequalities**

Let  $a, b, c,$  and  $d$  be real numbers.

**1. Transitive Property**

$$a < b \text{ and } b < c \implies a < c$$

**2. Addition of Inequalities**

$$a < b \text{ and } c < d \implies a + c < b + d$$

**3. Addition of a Constant**

$$a < b \implies a + c < b + c$$

**4. Multiplying by a Constant**

$$\text{For } c > 0, a < b \implies ac < bc$$

$$\text{For } c < 0, a < b \implies ac > bc \quad \text{Reverse the inequality symbol.}$$

Each of the properties above is true when the symbol  $<$  is replaced by  $\leq$  and  $>$  is replaced by  $\geq$ . For instance, another form of Property 3 is as follows.

$$a \leq b \implies a + c \leq b + c$$

The simplest type of inequality to solve is a **linear inequality** in one variable, such as  $x + 3 > 4$ . (For help with solving one-step linear inequalities, see Appendix E at this textbook's *Companion Website*.)

**EXAMPLE 1 Solving a Linear Inequality**

Solve  $5x - 7 > 3x + 9$ .

**Solution**

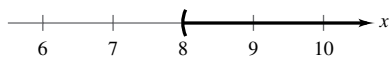
$$5x - 7 > 3x + 9 \quad \text{Write original inequality.}$$

$$2x - 7 > 9 \quad \text{Subtract } 3x \text{ from each side.}$$

$$2x > 16 \quad \text{Add } 7 \text{ to each side.}$$

$$x > 8 \quad \text{Divide each side by } 2.$$

The solution set is all real numbers that are greater than 8, which is denoted by  $(8, \infty)$ . The graph of this solution set is shown below. Note that a parenthesis at 8 on the real number line indicates that 8 is *not* part of the solution set.



**Solution Interval:**  $(8, \infty)$

**Explore the Concept**

Use a graphing utility to graph  $f(x) = 5x - 7$  and  $g(x) = 3x + 9$  in the same viewing window. (Use  $-1 \leq x \leq 15$  and  $-5 \leq y \leq 50$ .) For which values of  $x$  does the graph of  $f$  lie above the graph of  $g$ ? Explain how the answer to this question can be used to solve the inequality in Example 1.

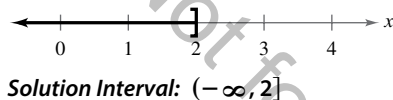
Note that the four inequalities forming the solution steps of Example 1 are all *equivalent* in the sense that each has the same solution set.

Checking the solution set of an inequality is not as simple as checking the solution(s) of an equation because there are simply too many  $x$ -values to substitute into the original inequality. However, you can get an indication of the validity of the solution set by substituting a few convenient values of  $x$ . For instance, in Example 1, try substituting  $x = 6$  and  $x = 10$  into the original inequality.

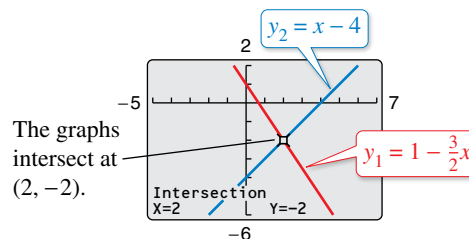
**EXAMPLE 2** Solving an InequalitySolve  $1 - \frac{3}{2}x \geq x - 4$ .**Algebraic Solution**

$$\begin{aligned} 1 - \frac{3}{2}x &\geq x - 4 && \text{Write original inequality.} \\ 2 - 3x &\geq 2x - 8 && \text{Multiply each side by the LCD.} \\ 2 - 5x &\geq -8 && \text{Subtract } 2x \text{ from each side.} \\ -5x &\geq -10 && \text{Subtract 2 from each side.} \\ x &\leq 2 && \text{Divide each side by } -5 \text{ and} \\ &&& \text{reverse the inequality symbol.} \end{aligned}$$

The solution set is all real numbers that are less than or equal to 2, which is denoted by  $(-\infty, 2]$ . The graph of this solution set is shown below. Note that a bracket at 2 on the number line indicates that 2 is part of the solution set.

**Graphical Solution**

Use a graphing utility to graph  $y_1 = 1 - \frac{3}{2}x$  and  $y_2 = x - 4$  in the same viewing window, as shown below.



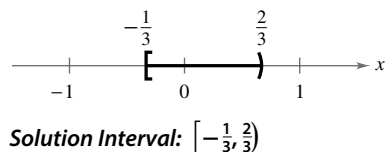
The graph of  $y_1$  lies above the graph of  $y_2$  to the left of their point of intersection,  $(2, -2)$ , which implies that  $y_1 \geq y_2$  for all  $x \leq 2$ .

Sometimes it is possible to write two inequalities as a **double inequality**, as demonstrated in Example 3.

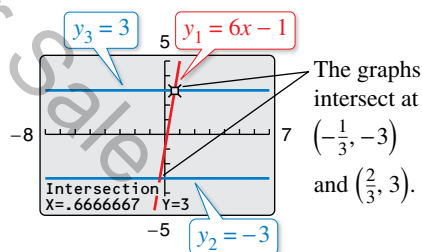
**EXAMPLE 3** Solving a Double InequalitySolve  $-3 \leq 6x - 1$  and  $6x - 1 < 3$ .**Algebraic Solution**

$$\begin{aligned} -3 &\leq 6x - 1 < 3 && \text{Write as a double inequality.} \\ -3 + 1 &\leq 6x - 1 + 1 < 3 + 1 && \text{Add 1 to each part.} \\ -2 &\leq 6x < 4 && \text{Simplify.} \\ \frac{-2}{6} &\leq \frac{6x}{6} < \frac{4}{6} && \text{Divide each part by 6.} \\ -\frac{1}{3} &\leq x < \frac{2}{3} && \text{Simplify.} \end{aligned}$$

The solution set is all real numbers that are greater than or equal to  $-\frac{1}{3}$  and less than  $\frac{2}{3}$ . The interval notation for this solution set is  $[-\frac{1}{3}, \frac{2}{3})$ . The graph of this solution set is shown below.

**Graphical Solution**

Use a graphing utility to graph  $y_1 = 6x - 1$ ,  $y_2 = -3$ , and  $y_3 = 3$  in the same viewing window, as shown below.



The graph of  $y_1$  lies above the graph of  $y_2$  to the right of  $(-\frac{1}{3}, -3)$  and the graph of  $y_1$  lies below the graph of  $y_3$  to the left of  $(\frac{2}{3}, 3)$ . This implies that  $y_2 \leq y_1 < y_3$  when  $-\frac{1}{3} \leq x < \frac{2}{3}$ .

## Inequalities Involving Absolute Values

### Solving an Absolute Value Inequality

Let  $x$  be a variable or an algebraic expression and let  $a$  be a positive real number.

1. The solutions of  $|x| < a$  are all values of  $x$  that lie between  $-a$  and  $a$ .

$$|x| < a \text{ if and only if } -a < x < a. \quad \text{Double inequality}$$

2. The solutions of  $|x| > a$  are all values of  $x$  that are less than  $-a$  or greater than  $a$ .

$$|x| > a \text{ if and only if } x < -a \text{ or } x > a. \quad \text{Compound inequality}$$

These rules are also valid when  $<$  is replaced by  $\leq$  and  $>$  is replaced by  $\geq$ .

### EXAMPLE 4 Solving Absolute Value Inequalities

Solve each inequality.

a.  $|x - 5| < 2$

b.  $|x - 5| > 2$

#### Algebraic Solution

a.  $|x - 5| < 2$

$$-2 < x - 5 < 2$$

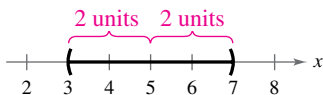
$$3 < x < 7$$

Write original inequality.

Write double inequality.

Add 5 to each part.

The solution set is all real numbers that are greater than 3 and less than 7. The interval notation for this solution set is  $(3, 7)$ . The graph of this solution set is shown below.



- b. The absolute value inequality  $|x - 5| > 2$  is equivalent to the following compound inequality:  $x - 5 < -2$  or  $x - 5 > 2$ .

Solve first inequality:  $x - 5 < -2$

$$x < 3$$

Write first inequality.

Add 5 to each side.

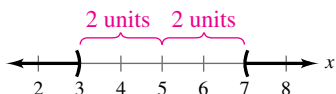
Solve second inequality:  $x - 5 > 2$

$$x > 7$$

Write second inequality.

Add 5 to each side.

The solution set is all real numbers that are less than 3 or greater than 7. The interval notation for this solution set is  $(-\infty, 3) \cup (7, \infty)$ . The symbol  $\cup$  is called a *union* symbol and is used to denote the combining of two sets. The graph of this solution set is shown below.

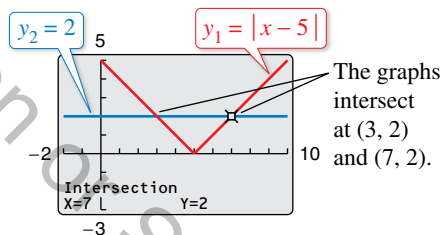


#### Graphical Solution

Use a graphing utility to graph

$$y_1 = |x - 5| \text{ and } y_2 = 2$$

in the same viewing window, as shown below.



- a. You can see that the graph of  $y_1$  lies below the graph of  $y_2$  when

$$3 < x < 7.$$

This implies that the solution set is all real numbers greater than 3 and less than 7.

- b. You can see that the graph of  $y_1$  lies above the graph of  $y_2$  when

$$x < 3$$

or when

$$x > 7.$$

This implies that the solution set is all real numbers that are less than 3 or greater than 7.

## Polynomial Inequalities

To solve a polynomial inequality such as  $x^2 - 2x - 3 < 0$ , use the fact that a polynomial can change signs only at its zeros (the  $x$ -values that make the polynomial equal to zero). Between two consecutive zeros, a polynomial must be entirely positive or entirely negative. This means that when the real zeros of a polynomial are put in order, they divide the real number line into intervals in which the polynomial has no sign changes. These zeros are the **key numbers** of the inequality, and the resulting open intervals are the **test intervals** for the inequality. For instance, the polynomial above factors as  $x^2 - 2x - 3 = (x + 1)(x - 3)$  and has two zeros,  $x = -1$  and  $x = 3$ , which divide the real number line into three test intervals:  $(-\infty, -1)$ ,  $(-1, 3)$ , and  $(3, \infty)$ . To solve the inequality  $x^2 - 2x - 3 < 0$ , you need to test only one value in each test interval.

### Finding Test Intervals for a Polynomial

To determine the intervals on which the values of a polynomial are entirely negative or entirely positive, use the following steps.

1. Find all real zeros of the polynomial, and arrange the zeros in increasing order. These zeros are the key numbers of the polynomial.
2. Use the key numbers to determine the test intervals.
3. Choose one representative  $x$ -value in each test interval and evaluate the polynomial at that value. If the value of the polynomial is negative, then the polynomial will have negative values for *every*  $x$ -value in the interval. If the value of the polynomial is positive, then the polynomial will have positive values for *every*  $x$ -value in the interval.

### EXAMPLE 5 Investigating Polynomial Behavior

To determine the intervals on which  $x^2 - 3$  is entirely negative and those on which it is entirely positive, factor the quadratic as  $x^2 - 3 = (x + \sqrt{3})(x - \sqrt{3})$ . The key numbers occur at  $x = -\sqrt{3}$  and  $x = \sqrt{3}$ . So, the test intervals for the quadratic are

$$(-\infty, -\sqrt{3}), (-\sqrt{3}, \sqrt{3}), \text{ and } (\sqrt{3}, \infty).$$

In each test interval, choose a representative  $x$ -value and evaluate the polynomial, as shown in the table.

Interval	$x$ -Value	Value of Polynomial	Sign of Polynomial
$(-\infty, -\sqrt{3})$	$x = -3$	$(-3)^2 - 3 = 6$	Positive
$(-\sqrt{3}, \sqrt{3})$	$x = 0$	$(0)^2 - 3 = -3$	Negative
$(\sqrt{3}, \infty)$	$x = 5$	$(5)^2 - 3 = 22$	Positive

The polynomial has negative values for every  $x$  in the interval  $(-\sqrt{3}, \sqrt{3})$  and positive values for every  $x$  in the intervals  $(-\infty, -\sqrt{3})$  and  $(\sqrt{3}, \infty)$ . In Figure B.40, the graph of  $y = x^2 - 3$  confirms this result.

### Technology Tip

Some graphing utilities will produce graphs of inequalities. For instance, you can graph  $2x^2 + 5x > 12$  by setting the graphing utility to *dot* mode and entering  $y = 2x^2 + 5x > 12$ . Using  $-10 \leq x \leq 10$  and  $-4 \leq y \leq 4$ , your graph should look like the graph shown below. The solution appears to be  $(-\infty, -4) \cup (\frac{3}{2}, \infty)$ . See Example 6 for an algebraic solution and for an alternative graphical solution.

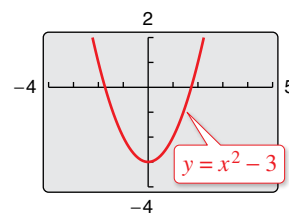
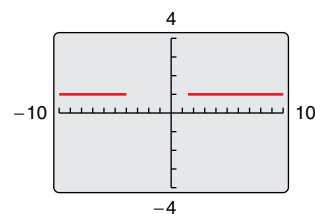


Figure B.40

To determine the test intervals for a polynomial inequality, the inequality must first be written in general form with the polynomial on one side and zero on the other.

### EXAMPLE 6 Solving a Polynomial Inequality

See *LarsonPrecalculus.com* for an interactive version of this type of example.

Solve  $2x^2 + 5x > 12$ .

#### Algebraic Solution

$$2x^2 + 5x - 12 > 0 \quad \text{Write inequality in general form.}$$

$$(x + 4)(2x - 3) > 0 \quad \text{Factor.}$$

$$\text{Key Numbers: } x = -4, x = \frac{3}{2}$$

$$\text{Test Intervals: } (-\infty, -4), (-4, \frac{3}{2}), (\frac{3}{2}, \infty)$$

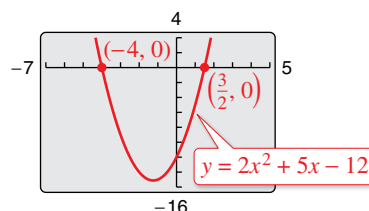
$$\text{Test: Is } (x + 4)(2x - 3) > 0?$$

After testing these intervals, you can see that the polynomial  $2x^2 + 5x - 12$  is positive on the open intervals  $(-\infty, -4)$  and  $(\frac{3}{2}, \infty)$ . So, the solution set of the inequality is

$$(-\infty, -4) \cup (\frac{3}{2}, \infty).$$

#### Graphical Solution

First write the polynomial inequality  $2x^2 + 5x > 12$  as  $2x^2 + 5x - 12 > 0$ . Then use a graphing utility to graph  $y = 2x^2 + 5x - 12$ . In the figure, you can see that the graph is *above* the  $x$ -axis when  $x$  is less than  $-4$  or when  $x$  is greater than  $\frac{3}{2}$ . So, you can graphically approximate the solution set to be  $(-\infty, -4) \cup (\frac{3}{2}, \infty)$ .



### EXAMPLE 7 Solving a Polynomial Inequality

$$2x^3 - 3x^2 - 32x + 48 > 0 \quad \text{Original inequality}$$

$$x^2(2x - 3) - 16(2x - 3) > 0 \quad \text{Factor by grouping.}$$

$$(x^2 - 16)(2x - 3) > 0 \quad \text{Distributive Property}$$

$$(x - 4)(x + 4)(2x - 3) > 0 \quad \text{Factor difference of two squares.}$$

The key numbers are  $x = -4$ ,  $x = \frac{3}{2}$ , and  $x = 4$ ; and the test intervals are  $(-\infty, -4)$ ,  $(-4, \frac{3}{2})$ ,  $(\frac{3}{2}, 4)$ , and  $(4, \infty)$ .

Interval	$x$ -Value	Polynomial Value	Conclusion
$(-\infty, -4)$	$x = -5$	$2(-5)^3 - 3(-5)^2 - 32(-5) + 48 = -117$	Negative
$(-4, \frac{3}{2})$	$x = 0$	$2(0)^3 - 3(0)^2 - 32(0) + 48 = 48$	Positive
$(\frac{3}{2}, 4)$	$x = 2$	$2(2)^3 - 3(2)^2 - 32(2) + 48 = -12$	Negative
$(4, \infty)$	$x = 5$	$2(5)^3 - 3(5)^2 - 32(5) + 48 = 63$	Positive

From this you can conclude that the polynomial is positive on the open intervals  $(-4, \frac{3}{2})$  and  $(4, \infty)$ . So, the solution set is  $(-4, \frac{3}{2}) \cup (4, \infty)$ .

When solving a polynomial inequality, be sure to account for the particular type of inequality symbol given in the inequality. For instance, in Example 7, the original inequality contained a “greater than” symbol and the solution consisted of two open intervals. If the original inequality had been  $2x^3 - 3x^2 - 32x + 48 \geq 0$ , the solution would have consisted of the closed interval  $[-4, \frac{3}{2}]$  and the interval  $[4, \infty)$ .

**EXAMPLE 8** Unusual Solution Sets

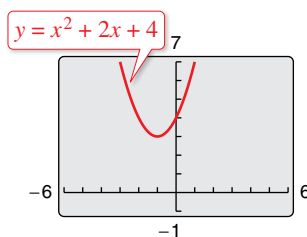
- a. The solution set of

$$x^2 + 2x + 4 > 0$$

consists of the entire set of real numbers,  $(-\infty, \infty)$ . In other words, the value of the quadratic

$$x^2 + 2x + 4$$

is positive for every real value of  $x$ , as shown in the figure at the right. (Note that this quadratic inequality has *no* key numbers. In such a case, there is only one test interval—the entire real number line.)



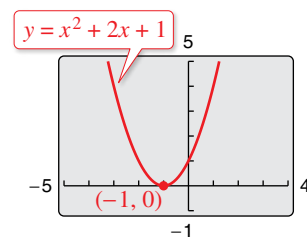
- b. The solution set of

$$x^2 + 2x + 1 \leq 0$$

consists of the single real number  $\{-1\}$ , because the quadratic

$$x^2 + 2x + 1$$

has one key number,  $x = -1$ , and it is the only value that satisfies the inequality, as shown in the figure at the right.



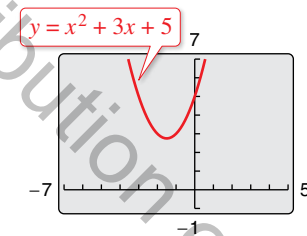
- c. The solution set of

$$x^2 + 3x + 5 < 0$$

is empty. In other words, the quadratic

$$x^2 + 3x + 5$$

is not less than zero for any value of  $x$ , as shown in the figure at the right.



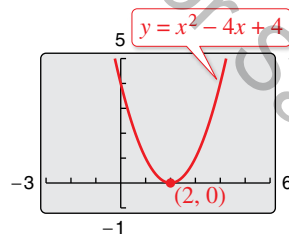
- d. The solution set of

$$x^2 - 4x + 4 > 0$$

consists of all real numbers *except* the number 2. In interval notation, this solution set can be written as  $(-\infty, 2) \cup (2, \infty)$ . The graph of

$$y = x^2 - 4x + 4$$

lies above the  $x$ -axis except at  $x = 2$ , where it touches the  $x$ -axis, as shown in the figure at the right.

**Technology Tip**

One of the advantages of technology is that you can solve complicated polynomial inequalities that might be difficult, or even impossible, to factor. For instance, you could use a graphing utility to approximate the solution of the inequality

$$x^3 - 0.2x^2 - 3.16x + 1.4 < 0.$$

## Rational Inequalities

The concepts of key numbers and test intervals can be extended to inequalities involving rational expressions. To do this, use the fact that the value of a rational expression can change sign only at its *zeros* (the  $x$ -values for which its numerator is zero) and its *undefined values* (the  $x$ -values for which its denominator is zero). These two types of numbers make up the *key numbers* of a rational inequality. When solving a rational inequality, begin by writing the inequality in general form with the rational expression on one side and zero on the other.

### EXAMPLE 9 Solving a Rational Inequality

Solve  $\frac{2x - 7}{x - 5} \leq 3$ .

#### Algebraic Solution

$$\frac{2x - 7}{x - 5} - 3 \leq 0 \quad \text{Write in general form.}$$

$$\frac{2x - 7 - 3x + 15}{x - 5} \leq 0 \quad \text{Write as single fraction.}$$

$$\frac{-x + 8}{x - 5} \leq 0 \quad \text{Simplify.}$$

Now, in standard form you can see that the key numbers are  $x = 5$  and  $x = 8$ , and you can proceed as follows.

*Key Numbers:*  $x = 5, x = 8$

*Test Intervals:*  $(-\infty, 5), (5, 8), (8, \infty)$

*Test:* Is  $\frac{-x + 8}{x - 5} \leq 0$ ?

Interval	$x$ -Value	Polynomial Value	Conclusion
$(-\infty, 5)$	$x = 0$	$\frac{-0 + 8}{0 - 5} = -\frac{8}{5}$	Negative
$(5, 8)$	$x = 6$	$\frac{-6 + 8}{6 - 5} = 2$	Positive
$(8, \infty)$	$x = 9$	$\frac{-9 + 8}{9 - 5} = -\frac{1}{4}$	Negative

By testing these intervals, you can determine that the rational expression  $(-x + 8)/(x - 5)$  is negative in the open intervals  $(-\infty, 5)$  and  $(8, \infty)$ . Moreover, because

$$\frac{-x + 8}{x - 5} = 0$$

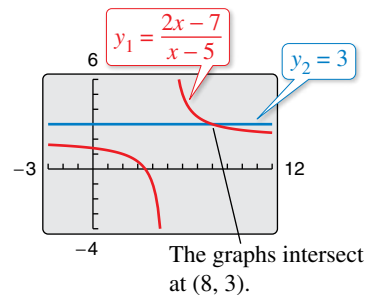
when  $x = 8$ , you can conclude that the solution set of the inequality is  $(-\infty, 5) \cup [8, \infty)$ .

#### Graphical Solution

Use a graphing utility to graph

$$y_1 = \frac{2x - 7}{x - 5} \quad \text{and} \quad y_2 = 3$$

in the same viewing window, as shown below.



The graph of  $y_1$  lies on or below the graph of  $y_2$  on the intervals  $(-\infty, 5)$  and  $[8, \infty)$ . So, you can graphically estimate the solution set to be all real numbers less than 5 or greater than or equal to 8.

Note in Example 9 that  $x = 5$  is not included in the solution set because the inequality is undefined when  $x = 5$ .



## Applications

In Section 1.2, you will study the *implied domain* of a function, the set of all  $x$ -values for which the function is defined. A common type of implied domain is used to avoid even roots of negative numbers, as shown in Example 10.

### EXAMPLE 10 Finding the Domain of an Expression

Find the domain of  $\sqrt{64 - 4x^2}$ .

#### Solution

Because  $\sqrt{64 - 4x^2}$  is defined only when  $64 - 4x^2$  is nonnegative, the domain is given by  $64 - 4x^2 \geq 0$ .

$$64 - 4x^2 \geq 0 \quad \text{Write in general form.}$$

$$16 - x^2 \geq 0 \quad \text{Divide each side by 4.}$$

$$(4 - x)(4 + x) \geq 0 \quad \text{Factor.}$$

The inequality has two key numbers:  $x = -4$  and  $x = 4$ . A test shows that  $64 - 4x^2 \geq 0$  in the *closed interval*  $[-4, 4]$ . The graph of

$$y = \sqrt{64 - 4x^2}$$

confirms that the domain is  $[-4, 4]$ , as shown in Figure B.41.

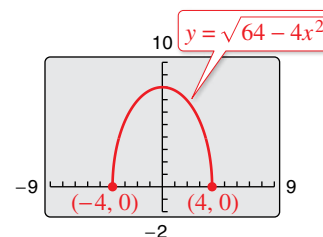


Figure B.41

### EXAMPLE 11 Height of a Projectile

A projectile is fired straight upward from ground level with an initial velocity of 384 feet per second. During what time period will the height of the projectile exceed 2000 feet?

#### Solution

The position of an object moving vertically can be modeled by the *position equation*

$$s = -16t^2 + v_0t + s_0$$

where  $s$  is the height in feet and  $t$  is the time in seconds. In this case,  $s_0 = 0$  and  $v_0 = 384$ . So, you need to solve the inequality  $-16t^2 + 384t > 2000$ . Using a graphing utility, graph

$$y_1 = -16t^2 + 384t \quad \text{and} \quad y_2 = 2000$$

as shown in Figure B.42. From the graph, you can determine that  $-16t^2 + 384t > 2000$  for  $t$  between approximately 7.6 and 16.4. You can verify this result algebraically.

$$-16t^2 + 384t > 2000 \quad \text{Write original inequality.}$$

$$t^2 - 24t < -125 \quad \text{Divide by } -16 \text{ and reverse inequality symbol.}$$

$$t^2 - 24t + 125 < 0 \quad \text{Write in general form.}$$

By the Quadratic Formula, the key numbers are  $t = 12 - \sqrt{19}$  and  $t = 12 + \sqrt{19}$ , or approximately 7.64 and 16.36. A test will verify that the height of the projectile will exceed 2000 feet when  $7.64 < t < 16.36$ —that is, during the time interval (7.64, 16.36) seconds. ■

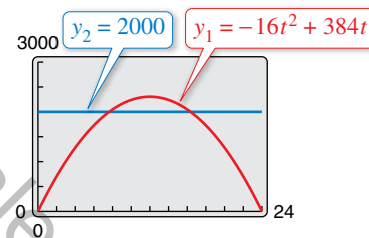


Figure B.42

## B.4 Exercises

See *CalcChat.com* for tutorial help and worked-out solutions to odd-numbered exercises. For instructions on how to use a graphing utility, see Appendix A.

### Vocabulary and Concept Check

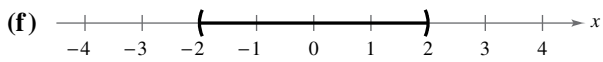
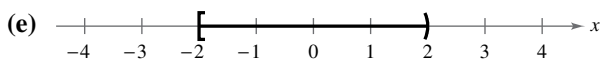
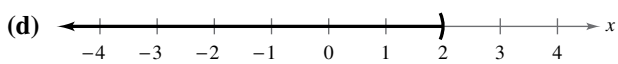
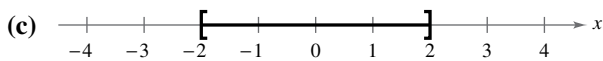
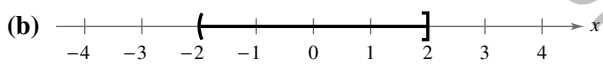
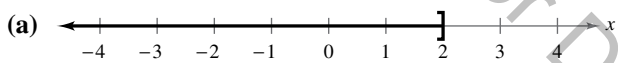
In Exercises 1–4, fill in the blank(s).

- It is sometimes possible to write two inequalities as one inequality, called a \_\_\_\_\_ inequality.
- The solutions of  $|x| \leq a$  are those values of  $x$  such that \_\_\_\_\_.
- The solutions of  $|x| \geq a$  are those values of  $x$  such that \_\_\_\_\_ or \_\_\_\_\_.
- The key numbers of a rational inequality are its \_\_\_\_\_ and its \_\_\_\_\_.
- Are the inequalities  $x - 4 < 5$  and  $x > 9$  equivalent?
- Which property of inequalities is shown below?

$$a < b \text{ and } b < c \quad \Rightarrow \quad a < c$$

### Procedures and Problem Solving

**Matching an Inequality with Its Graph** In Exercises 7–12, match the inequality with its graph. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]



- |                     |                        |
|---------------------|------------------------|
| 7. $x < 2$          | 8. $x \leq 2$          |
| 9. $-2 < x < 2$     | 10. $-2 < x \leq 2$    |
| 11. $-2 \leq x < 2$ | 12. $-2 \leq x \leq 2$ |

**Determining Solutions of an Inequality** In Exercises 13–16, determine whether each value of  $x$  is a solution of the inequality.

Inequality	Values	
13. $5x - 12 > 0$	(a) $x = 3$	(b) $x = -3$
	(c) $x = \frac{5}{2}$	(d) $x = \frac{3}{2}$
14. $-5 < 2x - 1 \leq 1$	(a) 2	(b) -2
	(c) 0	(d) $-\frac{1}{2}$
15. $-1 < \frac{3-x}{2} \leq 1$	(a) $x = -1$	(b) $x = \sqrt{5}$
	(c) $x = 1$	(d) $x = 5$
16. $ x - 10  \geq 3$	(a) $x = 13$	(b) $x = -1$
	(c) $x = 14$	(d) $x = 8$

**Solving an Inequality** In Exercises 17–30, solve the inequality and sketch the solution on the real number line. Use a graphing utility to verify your solution graphically.

- |                                   |                                |
|-----------------------------------|--------------------------------|
| 17. $6x > 42$                     | 18. $-10x \leq 40$             |
| 19. $4x + 7 < 3 + 2x$             | 20. $3x + 1 \geq 2 + x$        |
| 21. $2(1 - x) < 3x + 7$           | 22. $2x + 7 < 3(x - 4)$        |
| 23. $\frac{3}{4}x - 6 \leq x - 7$ | 24. $3 + \frac{2}{7}x > x - 2$ |
| 25. $1 \leq 2x + 3 \leq 9$        |                                |
| 26. $-8 \leq -3x + 5 < 13$        |                                |
| 27. $-8 \leq 1 - 3(x - 2) < 13$   |                                |
| 28. $0 \leq 2 - 3(x + 1) < 20$    |                                |
| 29. $-4 < \frac{2x - 3}{3} < 4$   |                                |
| 30. $0 \leq \frac{x + 3}{2} < 5$  |                                |

**Approximating a Solution** In Exercises 31–34, use a graphing utility to approximate the solution.

- $5 - 2x \geq 1$
- $20 < 6x - 1$
- $3(x + 1) < x + 7$
- $4(x - 3) > 8 - x$

**Approximating Solutions** In Exercises 35–38, use a graphing utility to graph the equation and graphically approximate the values of  $x$  that satisfy the specified inequalities. Then solve each inequality algebraically.

Equation	Inequalities	
35. $y = 2x - 3$	(a) $y \geq 1$	(b) $y \leq 0$
36. $y = \frac{2}{3}x + 1$	(a) $y \leq 5$	(b) $y \geq 0$
37. $y = -3x + 8$	(a) $-1 \leq y \leq 3$	(b) $y \leq 0$
38. $y = -\frac{1}{2}x + 2$	(a) $0 \leq y \leq 3$	(b) $y \geq 0$

**Solving an Absolute Value Inequality** In Exercises 39–46, solve the inequality and sketch the solution on the real number line. Use a graphing utility to verify your solutions graphically.

39.  $|5x| > 10$       40.  $\left|\frac{x}{2}\right| \leq 1$   
 41.  $|x - 7| \leq 6$       42.  $|x - 20| > 4$   
 43.  $\left|\frac{x - 3}{2}\right| \geq 5$       44.  $|x + 14| + 3 \geq 17$   
 45.  $10|1 - x| < 5$       46.  $3|4 - 5x| < 9$

**Approximating Solutions** In Exercises 47 and 48, use a graphing utility to graph the equation and graphically approximate the values of  $x$  that satisfy the specified inequalities. Then solve each inequality algebraically.

- | <i>Equation</i>                         | <i>Inequalities</i>           |
|-----------------------------------------|-------------------------------|
| 47. $y =  x - 3 $                       | (a) $y \leq 2$ (b) $y \geq 4$ |
| 48. $y = \left \frac{1}{2}x + 1\right $ | (a) $y \leq 4$ (b) $y \geq 1$ |

**Using Absolute Value Notation** In Exercises 49–56, use absolute value notation to define the interval (or pair of intervals) on the real number line.

- 49.
- 50.
- 51.
- 52.

53. All real numbers less than 10 units from 6  
 54. All real numbers no more than 8 units from  $-5$   
 55. All real numbers more than 3 units from  $-1$   
 56. All real numbers at least 5 units from 3

**Investigating Polynomial Behavior** In Exercises 57–62, determine the intervals on which the polynomial is entirely negative and those on which it is entirely positive.

57.  $x^2 - 4x - 5$       58.  $x^2 - 3x - 4$   
 59.  $2x^2 - 4x - 3$       60.  $-2x^2 + x + 5$   
 61.  $-x^2 + 6x - 10$       62.  $3x^2 + 8x + 6$

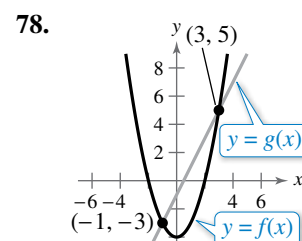
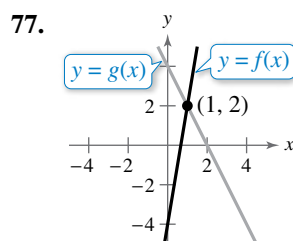
**Solving a Polynomial Inequality** In Exercises 63–76, solve the inequality and graph the solution on the real number line. Use a graphing utility to verify your solution graphically.

63.  $x^2 + 4x + 4 \geq 9$       64.  $x^2 - 6x + 9 < 16$   
 65.  $(x + 2)^2 < 25$       66.  $(x - 3)^2 \geq 1$   
 67.  $x^3 - 4x^2 \geq 0$       68.  $x^5 - 3x^4 \leq 0$

69.  $2x^3 + 5x^2 > 6x + 9$   
 70.  $2x^3 + 3x^2 < 11x + 6$   
 71.  $x^3 - 3x^2 - x > -3$   
 72.  $2x^3 + 13x^2 - 8x - 46 \geq 6$   
 73.  $3x^2 - 11x + 16 \leq 0$       74.  $4x^2 + 12x + 9 \leq 0$   
 75.  $4x^2 - 4x + 1 > 0$       76.  $x^2 + 3x + 8 > 0$

**Using Graphs to Find Solutions** In Exercises 77 and 78, use the graph of the function to solve the equation or inequality.

- (a)  $f(x) = g(x)$       (b)  $f(x) \geq g(x)$       (c)  $f(x) > g(x)$



**Approximating Solutions** In Exercises 79 and 80, use a graphing utility to graph the equation and graphically approximate the values of  $x$  that satisfy the specified inequalities. Then solve each inequality algebraically.

- | <i>Equation</i>                | <i>Inequalities</i>            |
|--------------------------------|--------------------------------|
| 79. $y = -x^2 + 2x + 3$        | (a) $y \leq 0$ (b) $y \geq 3$  |
| 80. $y = x^3 - x^2 - 16x + 16$ | (a) $y \leq 0$ (b) $y \geq 36$ |

**Solving a Rational Inequality** In Exercises 81–84, solve the inequality and graph the solution on the real number line. Use a graphing utility to verify your solution graphically.

81.  $\frac{1}{x} - x > 0$   
 82.  $\frac{1}{x} - 4 < 0$   
 83.  $\frac{x + 6}{x + 1} - 2 \leq 0$   
 84.  $\frac{x + 12}{x + 2} - 3 \geq 0$

**Approximating Solutions** In Exercises 85 and 86, use a graphing utility to graph the equation and graphically approximate the values of  $x$  that satisfy the specified inequalities. Then solve each inequality algebraically.

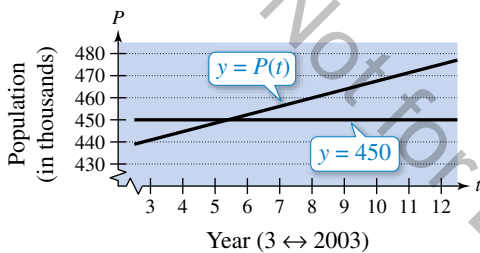
- | <i>Equation</i>              | <i>Inequalities</i>           |
|------------------------------|-------------------------------|
| 85. $y = \frac{3x}{x - 2}$   | (a) $y \leq 0$ (b) $y \geq 6$ |
| 86. $y = \frac{5x}{x^2 + 4}$ | (a) $y \geq 1$ (b) $y \leq 0$ |

**Finding the Domain of an Expression** In Exercises 87–92, find the domain of  $x$  in the expression.

87.  $\sqrt{x - 5}$                       88.  $\sqrt{6x + 15}$   
 89.  $\sqrt{-x^2 + x + 12}$             90.  $\sqrt{2x^2 - 8}$   
 91.  $\sqrt[4]{3x^2 - 20x - 7}$             92.  $\sqrt[4]{2x^2 + 4x + 3}$

**93. MODELING DATA**

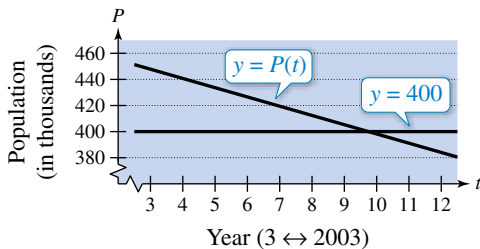
The graph models the population  $P$  (in thousands) of Sacramento, California, from 2003 through 2012, where  $t$  is the year, with  $t = 3$  corresponding to 2003. Also shown is the line  $y = 450$ . Use the graphs of the model and the horizontal line to write an equation or an inequality that could be solved to answer the question. Then answer the question. (Source: U.S. Census Bureau)



- (a) In what year did the population of Sacramento reach 450,000?  
 (b) During what time period was the population of Sacramento less than 450,000? greater than 450,000?

**94. MODELING DATA**

The graph models the population  $P$  (in thousands) of Cleveland, Ohio, from 2003 through 2012, where  $t$  is the year, with  $t = 3$  corresponding to 2003. Also shown is the line  $y = 400$ . Use the graphs of the model and the horizontal line to write an equation or an inequality that could be solved to answer the question. Then answer the question. (Source: U.S. Census Bureau)



- (a) In what year did the population of Cleveland reach 400,000?  
 (b) During what time period was the population of Cleveland less than 400,000? greater than 400,000?

**95. Height of a Projectile** A projectile is fired straight upward from ground level with an initial velocity of 160 feet per second.

- (a) At what instant will it be back at ground level?  
 (b) When will the height exceed 384 feet?

**96. Height of a Projectile** A projectile is fired straight upward from ground level with an initial velocity of 128 feet per second.

- (a) At what instant will it be back at ground level?  
 (b) When will the height be less than 128 feet?

**97. MODELING DATA**

The numbers  $D$  of doctorate degrees (in thousands) awarded to female students from 1991 through 2012 in the United States can be approximated by the model

$$D = 0.0743t^2 + 0.628t + 42.61, \quad 0 \leq t \leq 22$$

where  $t$  is the year, with  $t = 1$  corresponding to 1991. (Source: U.S. National Center for Education Statistics)

- (a) Use a graphing utility to graph the model.  
 (b) Use the *zoom* and *trace* features to find when the number of degrees was between 50 and 60 thousand.  
 (c) Algebraically verify your results from part (b).

**98. MODELING DATA**

You want to determine whether there is a relationship between an athlete's weight  $x$  (in pounds) and the athlete's maximum bench-press weight  $y$  (in pounds). Sample data from 12 athletes are shown below. (Spreadsheet at *LarsonPrecalculus.com*)

DATA
(165, 170), (184, 185), (150, 200),
(210, 255), (196, 205), (240, 295),
(202, 190), (170, 175), (185, 195),
(190, 185), (230, 250), (160, 150)

- (a) Use a graphing utility to plot the data.  
 (b) A model for the data is  $y = 1.3x - 36$ . Use the graphing utility to graph the equation in the same viewing window used in part (a).  
 (c) Use the graph to estimate the values of  $x$  that predict a maximum bench-press weight of at least 200 pounds.  
 (d) Use the graph to write a statement about the accuracy of the model. If you think the graph indicates that an athlete's weight is not a good indicator of the athlete's maximum bench-press weight, list other factors that might influence an individual's maximum bench-press weight.

**Why you should learn it** (p. B37) In Exercises 99–102, use the models below, which approximate the numbers of Bed Bath & Beyond stores  $B$  and Williams-Sonoma stores  $W$  for the years 2000 through 2013, where  $t$  is the year, with  $t = 0$  corresponding to 2000. (Sources: Bed Bath & Beyond, Inc.; Williams-Sonoma, Inc.)

**Bed Bath & Beyond:**

$$B = 86.5t + 342, 0 \leq t \leq 13$$

**Williams-Sonoma:**

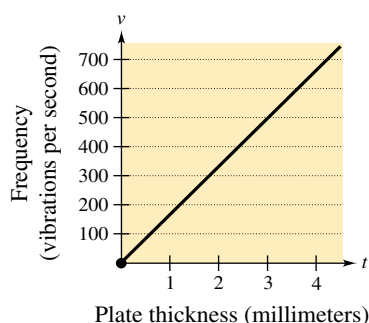
$$W = -2.92t^2 + 52.0t + 381, 0 \leq t \leq 13$$

99. Solve the inequality  $B(t) \geq 900$ . Explain what the solution of the inequality represents.
100. Solve the inequality  $W(t) \leq 600$ . Explain what the solution of the inequality represents.
101. Solve the equation  $B(t) = W(t)$ . Explain what the solution of the equation represents.
102. Solve the inequality  $B(t) \geq W(t)$ . Explain what the solution of the inequality represents.

**Music** In Exercises 103–106, use the following information. Michael Kasha, of Florida State University, used physics and mathematics to design a classical guitar. He used the model for the frequency of the vibrations on a circular plate

$$v = \frac{2.6t}{d^2} \sqrt{\frac{E}{\rho}}$$

where  $v$  is the frequency (in vibrations per second),  $t$  is the plate thickness (in millimeters),  $d$  is the diameter of the plate,  $E$  is the elasticity of the plate material, and  $\rho$  is the density of the plate material. For fixed values of  $d$ ,  $E$ , and  $\rho$ , the graph of the equation is a line, as shown in the figure.



103. Estimate the frequency when the plate thickness is 2 millimeters.
104. Estimate the plate thickness when the frequency is 600 vibrations per second.
105. Approximate the interval for the plate thickness when the frequency is between 200 and 400 vibrations per second.
106. Approximate the interval for the frequency when the plate thickness is less than 3 millimeters.

## Conclusions

**True or False?** In Exercises 107–109, determine whether the statement is true or false. Justify your answer.

107. If  $-10 \leq x \leq 8$ , then  $-10 \geq -x$  and  $-x \geq -8$ .

108. The solution set of the inequality

$$\frac{3}{2}x^2 + 3x + 6 \geq 0$$

is the entire set of real numbers.

109. The domain of  $\sqrt[3]{6-x}$  is  $(-\infty, 6]$ .

110. **Proof** The arithmetic mean of  $a$  and  $b$  is given by

$$\frac{a+b}{2}.$$

Order the statements of the proof to show that if  $a < b$ , then

$$a < \frac{a+b}{2} < b.$$

$$(i) a < \frac{a+b}{2} < b \quad (ii) 2a < 2b$$

$$(iii) 2a < a+b < 2b \quad (iv) a < b$$

111. **Think About It** Without performing any calculations, match the inequality with its solution. Explain your reasoning.

$$(a) 2x \leq -6 \quad (i) x \leq -8 \text{ or } x \geq 4$$

$$(b) -2x \leq 6 \quad (ii) x \geq -3$$

$$(c) |x+2| \leq 6 \quad (iii) -8 \leq x \leq 4$$

$$(d) |x+2| \geq 6 \quad (iv) x \leq -3$$

112. **Think About It** Consider the polynomial  $(x-a)(x-b)$  and the real number line shown below.



- (a) Identify the points on the line at which the polynomial is zero.
- (b) Determine the intervals on which the polynomial is entirely negative and those on which it is entirely positive. Explain your reasoning.
- (c) At what  $x$ -values does the polynomial change signs?

## B.5 Representing Data Graphically

### Line Plots

*Statistics* is the branch of mathematics that studies techniques for collecting, organizing, and interpreting data. In this appendix, you will study several ways to organize data. The first is a **line plot**, which uses a portion of a real number line to order numbers. Line plots are especially useful for ordering small sets of numbers (about 50 or less) by hand.

Many statistical measures can be obtained from a line plot. Two such measures are the *frequency* and *range* of the data. The **frequency** measures the number of times a value occurs in a data set. The **range** is the difference between the greatest and least data values. For example, consider the data values

$$20, 21, 21, 25, 32.$$

The frequency of 21 in the data set is 2 because 21 occurs twice. The range is 12 because the difference between the greatest and least data values is

$$32 - 20 = 12.$$

#### What you should learn

- ▶ Use line plots to order and analyze data.
- ▶ Use histograms to represent frequency distributions.
- ▶ Use bar graphs to represent and analyze data.
- ▶ Use line graphs to represent and analyze data.

#### Why you should learn it

Double bar graphs allow you to compare visually two sets of data over time. For example, in Exercises 15 and 16 on page B56, you are asked to estimate the difference in tuition between public and private institutions of higher education.

### EXAMPLE 1 Constructing a Line Plot

The scores from an economics class of 30 students are listed. The scores are for a 100-point exam. (*Spreadsheet at LarsonPrecalculus.com*)



93, 70, 76, 67, 86, 93, 82, 78, 83, 86, 64, 78, 76, 66, 83, 83, 96, 74, 69, 76, 64, 74, 79, 76, 88, 76, 81, 82, 74, 70

- a. Use a line plot to organize the scores.
- b. Which score occurs with the greatest frequency?
- c. What is the range of the scores?

#### Solution

- a. Begin by scanning the data to find the least and greatest numbers. For the data, the least number is 64 and the greatest is 96. Next, draw a portion of a real number line that includes the interval

$$[64, 96].$$

To create the line plot, start with the first number, 93, and record a • above 93 on the number line. Continue recording a • for each number in the list until you obtain the line plot shown in the figure.



- b. From the line plot, you can see that 76 occurs with the greatest frequency.
- c. Because the range is the difference between the greatest and least data values, the range of scores is

$$96 - 64 = 32.$$

## Histograms and Frequency Distributions

When you want to organize large sets of data, it is useful to group the data into intervals and plot the frequency of the data in each interval. A **frequency distribution** can be used to construct a **histogram**. A histogram uses a portion of a real number line as its horizontal axis. The bars of a histogram are not separated by spaces.

### EXAMPLE 2 Constructing a Histogram

The table at the right shows the percent of the resident population of each state and the District of Columbia who were at least 65 years old in 2012. Construct a frequency distribution and a histogram for the data. (Source: U.S. Census Bureau)

#### Solution

To begin constructing a frequency distribution, you must first decide on the number of intervals. There are several ways to group the data. However, because the least number is 8.5 and the greatest is 18.2, it seems that six intervals would be appropriate. The first would be the interval

$$[8, 10)$$

the second would be

$$[10, 12)$$

and so on. By tallying the data into the six intervals, you obtain the frequency distribution shown below.

Interval	Tally
[8, 10)	
[10, 12)	
[12, 14)	
[14, 16)	
[16, 18)	
[18, 20)	

You can construct the histogram by drawing a vertical axis to represent the number of states and a horizontal axis to represent the percent of the population 65 and older. Then, for each interval, draw a vertical bar whose height is the total tally, as shown in Figure B.43. Note that the break in the horizontal axis indicates that the numbers 0 through 7 have been omitted.

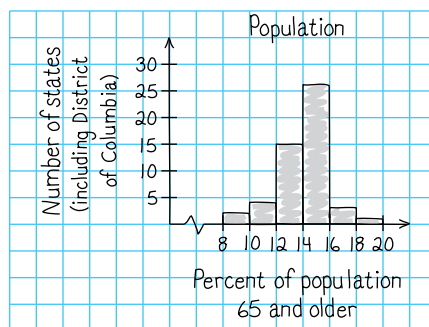


Figure B.43

DATA	
AK	8.5
AL	14.5
AR	15.0
AZ	14.8
CA	12.1
CO	11.8
CT	14.8
DC	11.4
DE	15.4
FL	18.2
GA	11.5
HI	15.1
IA	15.3
ID	13.2
IL	13.2
IN	13.6
KS	13.7
KY	14.0
LA	13.0
MA	14.5
MD	13.0
ME	17.0
MI	14.6
MN	13.6
MO	14.7
MS	13.5
MT	15.8
NC	13.8
ND	14.4
NE	13.8
NH	14.7
NJ	14.1
NM	14.1
NV	13.0
NY	14.1
OH	14.8
OK	14.1
OR	14.9
PA	16.0
RI	15.1
SC	14.7
SD	14.5
TN	14.3
TX	10.9
UT	9.5
VA	13.0
VT	15.7
WA	13.2
WI	14.4
WV	16.8
WY	13.0



## Bar Graphs

A **bar graph** is similar to a histogram, except that the bars can be either horizontal or vertical, and the labels of the bars are not necessarily numbers. For instance, the labels of the bars can be the months of a year, names of cities, or types of vehicles. Another difference between a bar graph and a histogram is that the bars in a bar graph are usually separated by spaces.

### EXAMPLE 3 Constructing a Bar Graph

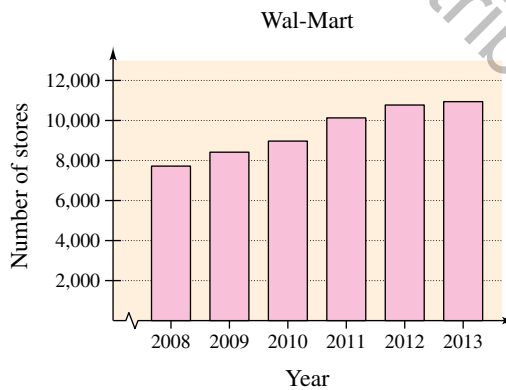
The table shows the numbers of Wal-Mart stores from 2008 through 2013. Construct a bar graph for the data. What can you conclude? (Source: Wal-Mart Stores, Inc.)

DATA	Year	Number of stores
	2008	7,720
	2009	8,416
	2010	8,970
	2011	10,130
	2012	10,773
	2013	10,942

Spreadsheet at LarsonPrecalculus.com

### Solution

To create a bar graph, begin by drawing a vertical axis to represent the number of stores and a horizontal axis to represent the year. The bar graph is shown below.



From the graph, you can see that the number of stores has steadily increased from 2008 to 2013. ■



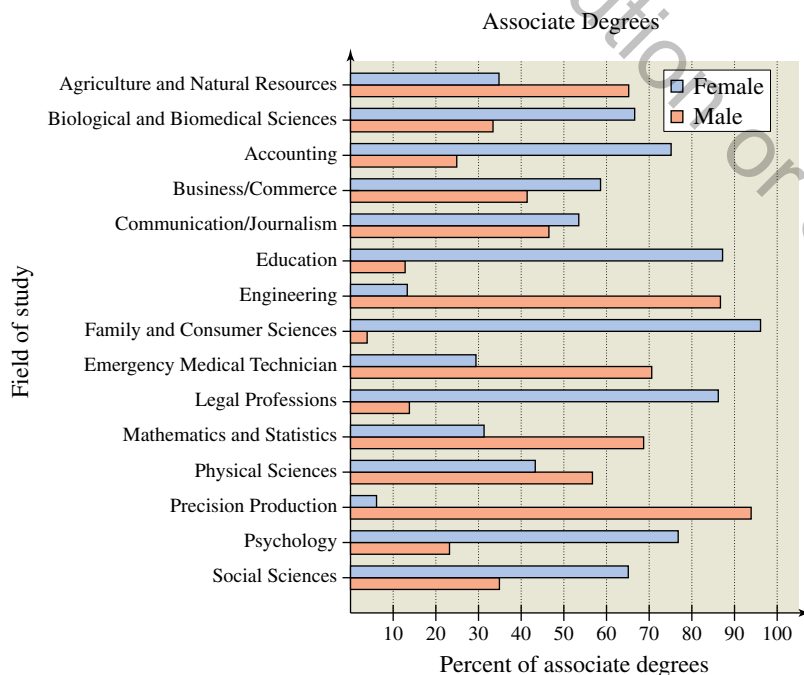
### EXAMPLE 4 Constructing a Double Bar Graph

The table shows the percents of associate degrees awarded to males and females for selected fields of study in the United States in 2012. Construct a double bar graph for the data. (Source: U.S. National Center for Education Statistics)

Field of Study	% Female	% Male
Agriculture and Natural Resources	34.8	65.2
Biological and Biomedical Sciences	66.6	33.4
Accounting	75.1	24.9
Business/Commerce	58.6	41.4
Communication/Journalism	53.5	46.5
Education	87.2	12.8
Engineering	13.3	86.7
Family and Consumer Sciences	96.1	3.9
Emergency Medical Technician	29.4	70.6
Legal Professions	86.2	13.8
Mathematics and Statistics	31.3	68.7
Physical Sciences	43.3	56.7
Precision Production	6.1	93.9
Psychology	76.8	23.2
Social Sciences	65.1	34.9

#### Solution

For the data, a horizontal bar graph seems to be appropriate. This makes it easier to label and read the bars, as shown below.



## Line Graphs

A **line graph** is similar to a standard coordinate graph. Line graphs are usually used to show trends over periods of time.

### EXAMPLE 5 Constructing a Line Graph

See *LarsonPrecalculus.com* for an interactive version of this type of example.

The table at the right shows the number of immigrants (in thousands) legally entering the United States for each decade from 1901 through 2010. Construct a line graph for the data. What can you conclude? (Source: U.S. Department of Homeland Security)

#### Solution

Begin by drawing a vertical axis to represent the number of immigrants in thousands. Then label the horizontal axis with decades and plot the points shown in the table. Finally, connect the points with line segments, as shown in Figure B.44. From the line graph, you can see that the number of immigrants hit a low point during the depression of the 1930s. Since then, the number has steadily increased.

Decade	Number (in thousands)
1901–1910	8,795
1911–1920	5,736
1921–1930	4,107
1931–1940	528
1941–1950	1,035
1951–1960	2,515
1961–1970	3,322
1971–1980	4,399
1981–1990	7,256
1991–2000	9,081
2001–2010	10,501

Spreadsheet at LarsonPrecalculus.com

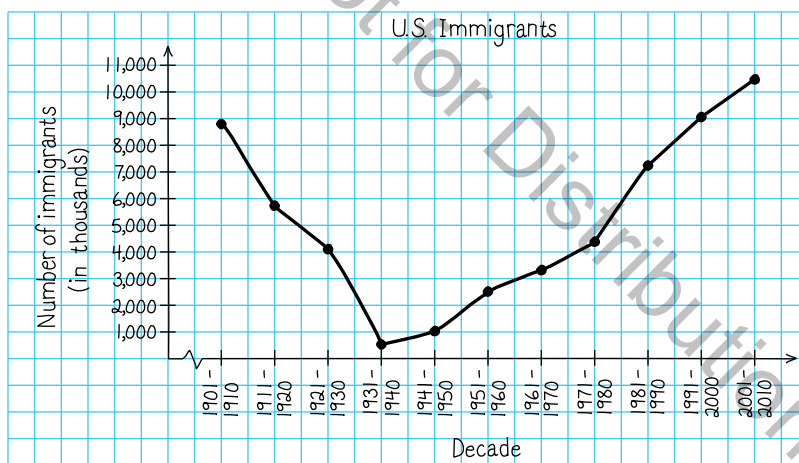


Figure B.44

You can use a graphing utility to check your sketch, as shown in Figure B.45.

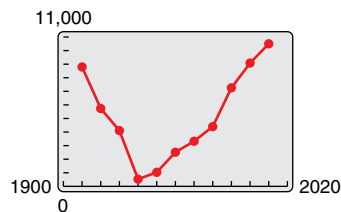


Figure B.45

### Technology Tip

You can use the *statistical plotting* feature of a graphing utility to create different types of graphs, such as line graphs. For instructions on how to use the *statistical plotting* feature, see Appendix A; for specific keystrokes, go to this textbook's *Companion Website*.

## B.5 Exercises

See *CalcChat.com* for tutorial help and worked-out solutions to odd-numbered exercises. For instructions on how to use a graphing utility, see Appendix A.

### Vocabulary and Concept Check

In Exercises 1 and 2, fill in the blank.

- \_\_\_\_\_ are useful for ordering small sets of numbers by hand.
- \_\_\_\_\_ show trends over periods of time.

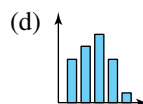
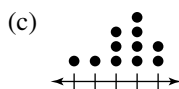
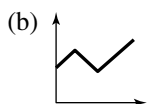
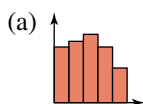
In Exercises 3–6, match the data display with its name.

3. Line plot

4. Bar graph

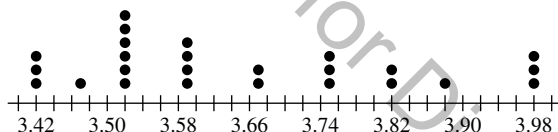
5. Histogram

6. Line graph



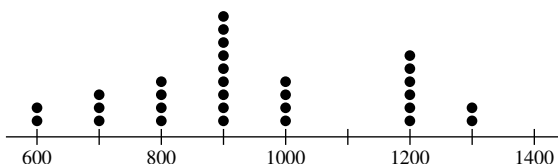
### Procedures and Problem Solving

7. **Economics** The line plot shows a sample of average prices of unleaded regular gasoline in 25 different cities.



- What price occurred with the greatest frequency?
- What is the range of prices?

8. **Waste Management** The line plot shows the weights (to the nearest hundred pounds) of municipal waste hauled by a garbage truck in 30 trips to a landfill.



- What weight occurred with the greatest frequency?
- What is the range of weights?

**Education** In Exercises 9 and 10, use the following scores from an algebra class of 30 students. The scores are for one 25-point quiz and one 100-point exam.

**Quiz** 20, 15, 14, 20, 16, 19, 10, 21, 24, 15, 15, 14, 15, 21, 19, 15, 20, 18, 18, 22, 18, 16, 18, 19, 21, 19, 16, 20, 14, 12

**Exam** 77, 100, 77, 70, 83, 89, 87, 85, 81, 84, 81, 78, 89, 78, 88, 85, 90, 92, 75, 81, 85, 100, 98, 81, 78, 75, 85, 89, 82, 75

- Construct a line plot for the quiz. Which score(s) occurred with the greatest frequency?
- Construct a line plot for the exam. Which score(s) occurred with the greatest frequency?

11. **Demographics** The list shows the percents of individuals living below the poverty level in the 50 states in 2012. Use a frequency distribution and a histogram to organize the data. (*Spreadsheet at LarsonPrecalculus.com*) (Source: U.S. Census Bureau)



AK 10.8	AL 15.8	AR 19.4	AZ 18.1
CA 16.4	CO 12.5	CT 10.2	DE 13.6
FL 15.1	GA 18.3	HI 13.0	IA 10.4
ID 15.1	IL 13.4	IN 15.4	KS 14.2
KY 16.9	LA 21.1	MA 10.9	MD 9.6
ME 13.1	MI 14.3	MN 10.0	MO 15.3
MS 19.7	MT 15.0	NC 16.3	ND 10.7
NE 11.2	NH 7.9	NJ 10.4	NM 21.3
NV 15.6	NY 16.6	OH 15.2	OK 16.0
OR 13.9	PA 13.2	RI 13.5	SC 17.8
SD 13.7	TN 17.4	TX 17.2	UT 11.0
VA 11.0	VT 11.4	WA 12.1	WI 12.2
WV 17.1	WY 10.2		

12. **Education** The list shows the numbers of students enrolled in public schools (in thousands) in the 50 states and the District of Columbia in 2012. Use a frequency distribution and a histogram to organize the data. (*Spreadsheet at LarsonPrecalculus.com*) (Source: National Education Association)

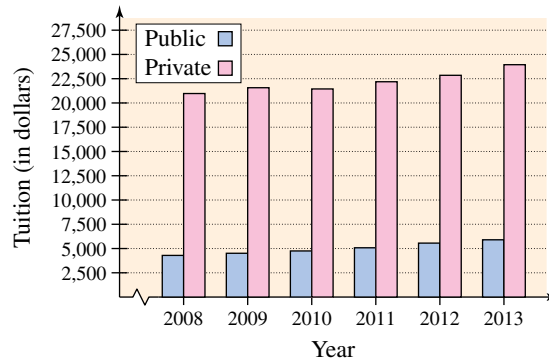


AK 128	AL 736	AR 472	AZ 1070
CA 6185	CO 869	CT 550	DC 76
DE 131	FL 2681	GA 1703	HI 177
IA 501	ID 294	IL 2084	IN 1031
KS 486	KY 659	LA 708	MA 954
MD 860	ME 185	MI 1544	MN 843
MO 907	MS 493	MT 143	NC 1488
ND 99	NE 303	NH 189	NJ 1358
NM 334	NV 478	NY 2590	OH 1868
OK 673	OR 564	PA 1739	RI 134
SC 683	SD 127	TN 980	TX 5059
UT 601	VA 1266	VT 82	WA 1053
WI 872	WV 282	WY 91	

13. **Meteorology** The table shows the normal monthly precipitation (in inches) in Seattle, Washington. Construct a bar graph for the data. Write a brief statement regarding the precipitation in Seattle, Washington, throughout the year. (Source: National Oceanic and Atmospheric Administration)

Month	Precipitation
January	5.47
February	3.52
March	3.85
April	2.79
May	2.01
June	1.57
July	0.85
August	0.94
September	1.30
October	3.38
November	5.98
December	6.06

- Why you should learn it* (p. B50) In Exercises 15 and 16, the double bar graph shows the mean tuitions (in dollars) charged by public and private institutions of higher education in the United States from 2008 through 2013. (Source: U.S. National Center for Education Statistics)



14. **Business** The table shows the revenues (in billions of dollars) for Costco Wholesale stores in the years from 1997 through 2013. Construct a bar graph for the data. Write a brief statement regarding the revenue of Costco Wholesale stores over time. (Source: Costco Wholesale Corporation)

Year	Revenue (in billions of dollars)
1997	21.874
1998	24.270
1999	27.456
2000	32.164
2001	34.797
2002	38.762
2003	42.546
2004	48.107
2005	52.935
2006	60.151
2007	64.909
2008	72.483
2009	71.449
2010	77.946
2011	88.915
2012	99.137
2013	105.156

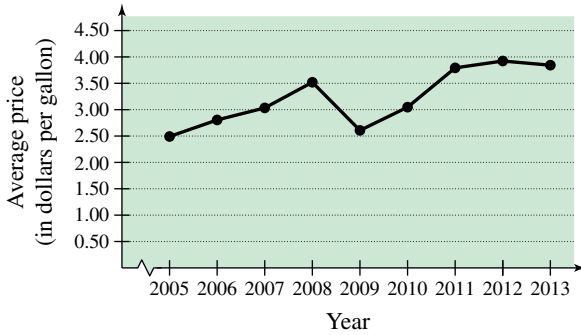
15. Approximate the difference in tuition charges for public and private schools for each year.  
 16. Approximate the increase in tuition charges for each type of institution from year to year.  
 17. **Education** The table shows the total college enrollments (in thousands) for women and men in the United States from 2006 through 2012. Construct a double bar graph for the data. (Source: U.S. National Center for Education Statistics)

Year	Women (in thousands)	Men (in thousands)
2006	10,184	7,575
2007	10,432	7,816
2008	10,914	8,189
2009	11,658	8,770
2010	11,971	9,045
2011	11,968	9,026
2012	11,724	8,919

18. **Demographics** The table shows the populations (in millions) of the five most populous cities in the United States in 1990 and 2012. Construct a double bar graph for the data. (Source: U.S. Census Bureau)

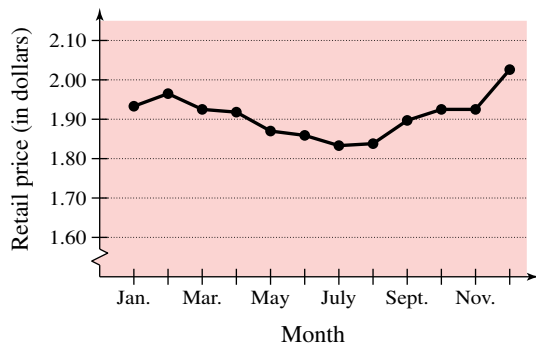
City	1990 population (in millions)	2012 population (in millions)
New York, NY	7.32	8.34
Los Angeles, CA	3.49	3.86
Chicago, IL	2.78	2.71
Houston, TX	1.63	2.16
Philadelphia, PA	1.59	1.55

**Economics** In Exercises 19–22, use the line graph, which shows the average prices of a gallon of premium unleaded gasoline from 2005 through 2013. (Source: U.S. Energy Information Administration)



19. Describe the trend in the price of premium unleaded gasoline from 2005 to 2008.
20. Describe the trend in the price of premium unleaded gasoline from 2008 to 2009.
21. Approximate the percent decrease in the price per gallon of premium unleaded gasoline from 2008 to 2010.
22. Approximate the percent increase in the price per gallon of premium unleaded gasoline from 2009 to 2013.

**Agriculture** In Exercises 23–26, use the line graph, which shows the average retail price (in dollars) of one dozen Grade A large eggs in the United States for each month in 2013. (Source: U.S. Bureau of Labor Statistics)



23. What is the highest price of one dozen Grade A large eggs shown in the graph? When did this price occur?
24. What was the difference between the highest price and the lowest price of one dozen Grade A large eggs in 2013?
25. Determine when the price of eggs showed the greatest rate of increase from one month to the next.

26. Describe any trends shown by the line graph. Then predict the average price of eggs in February of 2014. Are you confident that your prediction is within \$0.10 of the actual average retail price? Explain your reasoning.

**27. Human Resources** The table shows the percents of wives in two-income families who earned more than their husbands in the United States from 2000 through 2012. Construct a line graph for the data. Write a brief statement describing what the graph reveals. (Source: U.S. Bureau of Labor Statistics)

Year	Percent
2000	23.3
2001	24.1
2002	25.0
2003	25.2
2004	25.3
2005	25.5
2006	25.7
2007	25.9
2008	26.6
2009	28.9
2010	29.2
2011	28.1
2012	29.0

**28. Economics** The table shows the trade deficits (the differences between imports and exports) of the United States from 2004 through 2013. Construct a line graph for the data. Write a brief statement describing what the graph reveals. (Source: U.S. Census Bureau)

Year	Trade deficit (in billions of dollars)
2004	655
2005	772
2006	828
2007	809
2008	816
2009	504
2010	635
2011	725
2012	731
2013	689

29. **Marketing** The table shows the costs (in millions of dollars) of a 30-second television advertisement during the Super Bowl from 1999 through 2014. Use a graphing utility to construct a line graph for the data. (Source: superbowl-ads.com)

Year	Cost (in millions of dollars)
1999	1.60
2000	2.10
2001	2.05
2002	1.90
2003	2.10
2004	2.25
2005	2.40
2006	2.50
2007	2.60
2008	2.70
2009	3.00
2010	2.65
2011	3.00
2012	3.50
2013	3.80
2014	4.00

31. **Athletics** The table shows the numbers of participants (in thousands) in high school athletic programs in the United States from 2002 through 2013. Organize the data in an appropriate display. Explain your choice of graph. (Source: National Federation of State High School Associations)

Year	Female athletes (in thousands)	Male athletes (in thousands)
2002	2807	3961
2003	2856	3989
2004	2865	4038
2005	2908	4110
2006	2953	4207
2007	3022	4321
2008	3057	4372
2009	3114	4423
2010	3173	4456
2011	3174	4494
2012	3208	4485
2013	3223	4491

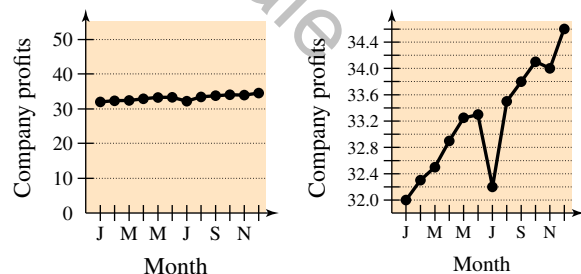
30. **Networking** The list shows the percent of households in each of the 50 states and the District of Columbia with Internet access in 2012. Use a graphing utility to organize the data in the graph of your choice. Explain your choice of graph. (Spreadsheet at LarsonPrecalculus.com) (Source: U.S. Census Bureau)

State	Percent	State	Percent	State	Percent
AK	83.1	AL	72.0	AR	74.8
AZ	73.7	CA	81.3	CO	85.7
CT	84.0	DC	77.5	DE	78.2
FL	81.5	GA	79.2	HI	85.0
IA	80.2	ID	86.8	IL	82.7
IN	77.7	KS	80.7	KY	74.4
LA	67.7	MA	86.0	MD	83.6
ME	84.2	MI	80.8	MN	87.4
MO	75.4	MS	64.8	MT	78.3
NC	77.7	ND	84.0	NE	76.1
NH	87.7	NJ	83.7	NM	73.9
NV	82.8	NY	81.0	OH	75.6
OK	74.8	OR	87.9	PA	80.8
RI	82.2	SC	73.7	SD	78.3
TN	74.3	TX	72.6	UT	85.2
VA	77.2	VT	85.7	WA	85.1
WI	80.7	WV	70.6	WY	82.9

**Conclusions**

32. **Writing** Describe the differences between a bar graph and a histogram.
33. **Writing** Describe the differences between a line plot and a line graph.
34. **Writing** The graphs shown below represent the same data points.

- (a) Which of the two graphs is misleading and why?
- (b) Discuss other ways in which graphs can be misleading.
- (c) Why would it be beneficial for someone to use a misleading graph?



35. **Think About It** How can you decide which type of graph to use when you are organizing data?